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DENSE GRANULAR FLOWS : FRICTION AND JAMMING

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ABSTRACT

Using discrete numerical simulations, we discuss the constitutive law and the jamming mechanisms of dense granular flows (dry or cohesive grains), both at the macroscopic level (friction and dilatancy laws) and at the level of the contact network (coordination number and mobilization of friction). Among the characteristics of the grains (friction and restitution coefficients, rigidity, interfacial energy) and of the shear (pressure and shear rate), our study identifies the dimensionless numbers which determine the various flow regimes. We first study the homogeneous case of plane shear flows and then describe the influence of the wall roughness in the case of inclined plane flows.

Keywords : Granular materials, dense flows, jamming transition, friction, internal state variables

INTRODUCTION

Of interest both in geophysics and for industrial processes, dense granular flows, close to the jamming transition, are still not well understood (Pouliquen & Chevoir, 2002; GDR, 2004). Complementary to physical experiments with model materials, discrete numerical simulations give access to detailed quantities inside the flow (kinematics of the grains, contact network) from which it is possible to measure average and fluctuating quantities such as solid fraction, velocity and stress component profiles. From those quantities, it is possible to deduce the constitutive laws in velocity controlled simulations, and the jamming mechanisms in stress controlled simulations. In the following, we shall first describe the simulated systems and the simulation methods. Then, we will discuss the constitutive law in the simplest case of an homogeneous shear. Last, we will discuss the influence of the wall roughness and the jamming transition for inclined plane flows.

SIMULATED SYSTEMS AND SIMULATION METHODS

Our simulated granular material is an assembly of n (between 1000 and 5000) slightly polydispersed disks of average diameter d (± 20%) and mass m. Plane shear flows and jamming down an inclined plane have been simulated by the molecular dynamics (MD) method (da Cruz, 2004). Then, the normal force F_N is a function of the normal deformation of the contact h, sum of a linear elastic term and a viscous dissipative term : $F_N = K_N h + \alpha_N dh/dt$. The tangential force F_T is a function of the relative tangential displacement δ : $dF_T/d\delta = K_T$, with the Coulomb condition $|F_T| < \mu F_N$. μ is the microscopic friction coefficient. The ratio of normal and tangential stiffness K_N / K_T is chosen equal to 2. The value of the viscous damping α_N is related to the value of the coefficient of normal restitution e_N in binary collisions : $\alpha_N = -2\,(mK_N)^{1/2}\, ln(e_N) / (\pi^2 + ln^2(e_N))^{1/2}$. The effect of adhesion, significant in powders, is modeled by adding a normal attractive force proportional to the surface of the contact : $F^C = -\gamma\,(dh)^{1/2}$, where γ is the interfacial energy (Mattutis & Schinner, 2001). Then, the traction resistance between two contacting grains is equal to $\gamma^2 d/4K_N$. Steady flows down an inclined plane have been simulated by the contact dynamics (CD) method (Chevoir *et al.* 2001; Prochnow, 2002). This method (Moreau, 1994) deals with rigid grains, treats maintained contacts and multiple

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collisions on the same level, and uses three macroscopic coefficients (normal and tangential restitution e_N and e_T, friction μ). If not otherwise precised, μ is equal to 0.4 in both simulation methods, e_N = 0.1 in MD and $e_N = e_T = 0$ in CD.

The flow is confined by one (inclined plane) or two (plane shear) rough walls. The wall roughness is made of contiguous grains, similar to the flowing ones. Periodic boundary conditions are applied along the flow direction. The flows are simulated inside a window of length L (usually 40 d). The system fluctuates in space and time, but once the flow is steady and uniform, we average over space (along the flow direction x) and time. We measure the average profiles of solid fraction, shear rate and stress components (pressure P, shear stress S, normal stresses are equal), as well as micromechanical quantities : coordination number z^* and mobilization of friction M (proportion of sliding contacts).

Dimensional analysis

Apart from the two local parameters describing interactions between grains (e_N and μ), the system is characterized by three dimensionless numbers. The *rigidity number* $\kappa = K_N/P$ describes the deformation (h/d) of the grains submitted to the confining pressure P. In this study, we stay in the limit of rigid grains (κ> 400). The *inertial number* $I = \dot{\gamma}\sqrt{m/Pd}$ compares the inertial stress to the confining pressure P. The *cohesion number* $\eta = \gamma^2/K_N P$ describes the intensity of the cohesion for a given confining pressure P.

HOMOGENEOUS SHEAR

We first discuss the simplest shear geometry (plane shear without gravity) where the stress distribution is homogeneous (Fig.1) (da Cruz, 2004). The material is sheared between two parallel walls distant of H (between 20 and 100 d). One of the wall is fixed and the other moves with a controlled velocity V. The pressure P is controlled through the dilatancy of the material. At each time step, the normal velocity v_n of the moving wall is given by the normal force N exerted by the grains on the wall by : $v_n = (PL-N)/g_p$, where g_p is a viscous damping parameter, so that equilibrium is obtained when $P = N/L$. This introduces another dimensionless number $g_p/(mK_N)^{1/2}$, which remains small in our simulations, that is to say the timescale of the fluctuations of H is imposed by the material rather than the walls which stick to the material.

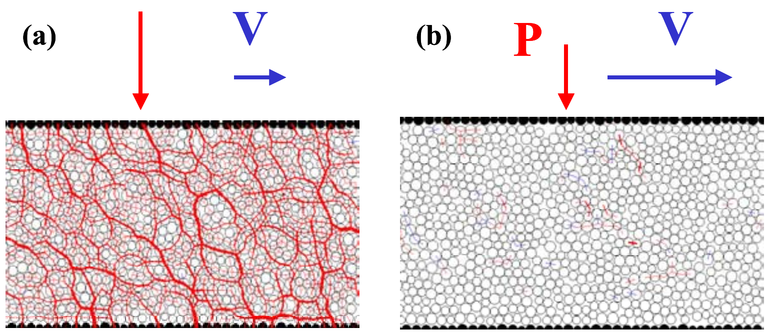

Figure 1 : Plane shear (a) quasi-static regime (*I* = 0.01) (b) Dynamic regime (*I* = 0.2).

The steady state does not depend on the preparation of the system (loose state – random sort, or dense state - cyclic compaction of frictionless grains), and the results are identical to fixed volume simulations (da Cruz *et al.*, 2003). Except for monodispersed systems and soft grains, we do not observe shear localisation, but obtain an homogeneous well controlled shear state (pressure P and shear rate $\dot{\gamma}$). The steady flows are characterized by two macroscopic quantities, which are averaged in the central part of the cell, excluding the five first layers near the walls : the solid fraction Φ and the macroscopic friction $\mu^* = S/P$, ratio of the shear stress and the pressure.

Dry grains

Dimensional analysis has shown that, in the limit of rigid and dry grains, the shear state is defined by the single inertial number I. The quasi-static regime ($I < 0.01$) is characterized by a dense network of maintained contacts (Fig.1a). It corresponds to the critical state in soil mechanics with a maximum solid fraction Φ_m and a macroscopic friction μ_S^*. When I increases, that is to say when the shear rate increases and/or the pressure decreases, the material slightly dilates, the coordination number decreases (Fig.3a) and the proportion of collisions increases up to a dynamic purely collisional regime ($I > 0.2$) (Fig.1b). The transition between those two regimes (intermediate regime : $0.01 < I < 0.2$) is progressive. Correlatively, the evolution of the mobilisation of friction (Fig.3b) leads to an increase of the macroscopic friction.

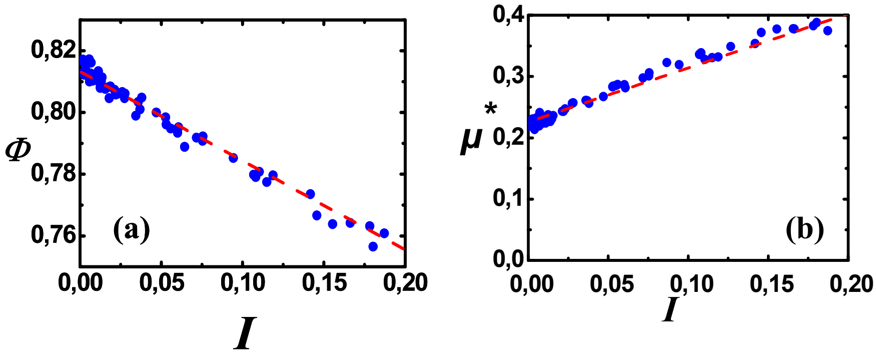

Figure 2 : Plane shear ($\mu \neq 0$) – in red linear fit (a) Dilatancy law (b) Friction law.

The variations with I of the solid fraction ("dilatancy law") and of the macroscopic friction ("friction law"), shown in Fig.2, reveal the following simple dependencies (with a ≈ 1 and b ≈ 0,3) :

$$\begin{cases} \Phi(I) = \Phi_{\mathrm{m}} - bI \\ \mu^*(I) = \mu_S^* + aI \end{cases} \tag{1}$$

From those two laws, we identify the constitutive law in the intermediate regime, which is an interesting information in the present debates (hydrodynamic models inspired by the glass transition, frictional-collisional models…(Pouliquen & Chevoir, 2002; GDR, 2004)). This constitutive law is viscoplastic, with a Coulomb threshold and viscous stresses which depend on the square of the shear rate and diverge near the maximum solid fraction :

$$\begin{cases} P = b^2 m / d \dfrac{\dot{\gamma}^2}{(\Phi_m - \Phi)^2} \\ S = \mu_S^* P + abm / d \dfrac{\dot{\gamma}^2}{(\Phi_m - \Phi)} \end{cases} \tag{2}$$

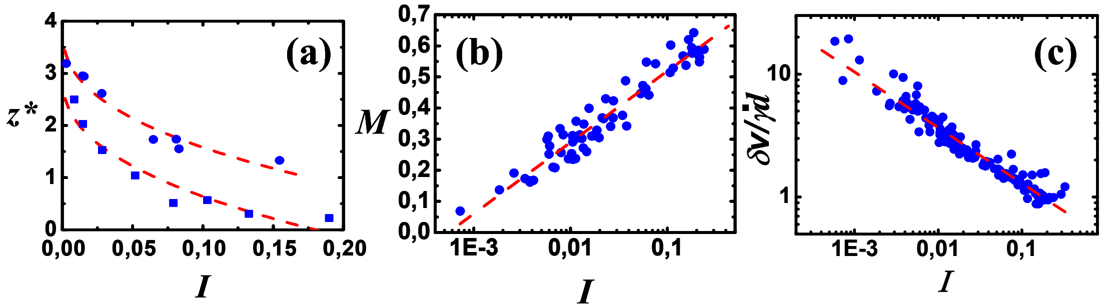

Figure 3 : Plane shear (a) Coordination number (μ = 0 – blue disks: e_N = 0.1 – blue squares: e_N = 0.9) (b) Mobilisation of friction (c) Relative fluctuations of the translation velocity.

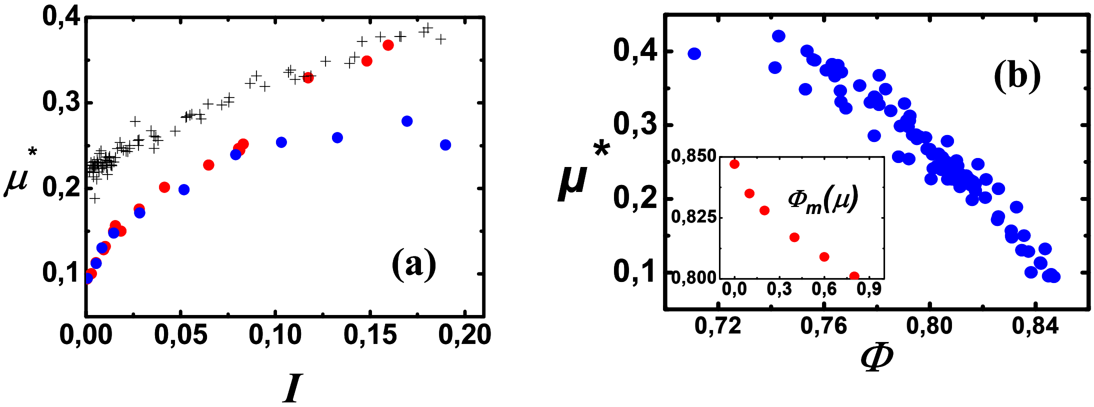

Figure 4 : Plane shear (a) Influence of μ and e_N (crosses : $\mu \neq 0$, red disks : $\mu = 0$ and $e_N = 0.1$, blue disks: $\mu = 0$ and $e_N = 0.9$) (b) Friction law as a function of solid fraction. Insert, maximum solid fraction as a function of μ.

In the dynamic regime, the constitutive law depends on the restitution coefficient (Fig.4a). But in the intermediate regime, the dilatancy and friction laws are nearly independent of the mechanical characteristics of the grains (restitution anf friction coefficient, rigidity). However, in the case of frictionless grains, the friction law keeps the same shape but is shifted to lower values (μ_S^* decreases from 0.2 to 0.1) (Fig.4a). It appears that the maximum solid fraction is not a purely geometrical quantity, since it decreases with the microscopic friction μ (Fig.4b-insert). The $\mu*(\Phi)$ graph, gathering data for various μ, reveals a single master curve (Fig.4b). It is noteworthy that a small variation of Φ (10%) is responsible of a variation of $\mu*$ by a factor 4.

In the quasi-static regime, when $I \to 0$, the relative fluctuations of the translation velocity $\delta v / \dot{\gamma} d$ increase according to a power law (Fig.3c), whereas the relative fluctuations of the stress components decrease. The flows, which are steady and continuous for $I > 0.001$, become intermittent for smaller I. Those intermittencies seem associated to correlated motions of blocks, which have motivated various non-local rheological models (Mills *et al.,* 1999; Pouliquen *et al.,* 2001; Ertas & Halsey, 2002).

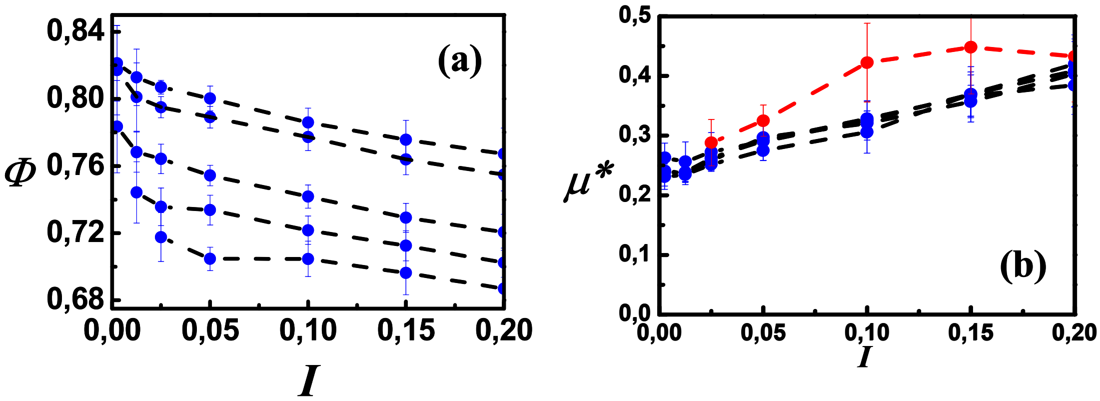

Figure 5 : Plane shear – cohesive grains (η = 0, 0.25, 4, 12, 25). The barrs correspond to the fluctuations inside the layer (a) Dilatancy law (b) Friction law – in red η = 25.

Cohesive grains

The simple adhesion model has been used to study the influence of cohesion (Rognon *et al.*, 2004). Then, in the limit of rigid grains, the flow regime should only depend on the inertial number I and on the cohesion number η. We observe a transition between two flow regimes, steady (inertial and/or small cohesion) and intermittent (quasi-static and/or strong cohesion). For $\eta < 12$, the friction law (1) is not modified, and the dilatancy law remains qualitatively similar to the one measured for dry grains (1), except that the maximum solid fraction strongly decreases with the cohesion (Fig.5a). For a strong enough cohesion ($\eta > 12$), we do not observe steady flows anymore in the explored range of I. The macroscopic friction increases significantly (Fig.5b-red curve), as well as fluctuations. The critical value of cohesion is in agreement with the one anticipated from dimensional analysis. In parallel to the decrease of the solid fraction, we observe an increase of the coordination number and an homogeneisation of the distribution of the contact directions. This is the sign of an organization of the grains in clusters separated by voids. The proportion of void does not depend on I, but increases linearly with η. Moreover, the cohesion increases the lifetime of contacts. When all the contacts become persistent, the material is made of a single rigid block which sticks alternatively to one of the two walls.

INCLINED PLANE

In the presence of an heterogeneous distribution of the stress components (plane shear with gravity, annular shear cell (da Cruz, 2004)), it has been shown that the dilatancy and friction laws remain qualitatively the same, but the variation of the stress ratio S/P leads to a localisation of the shear. We now focus on the case of dense flows of dry grains down a rough inclined plane (Silbert et al., 2001; Chevoir *et al*., 2001; Prochnow, 2002; da Cruz, 2004). We want to show how the friction law is modified, since this quantity is crucial for the prediction of the spread of a granular mass down a slope, in the frame of a depth averaged decription (Pouliquen & Forterre, 2002).

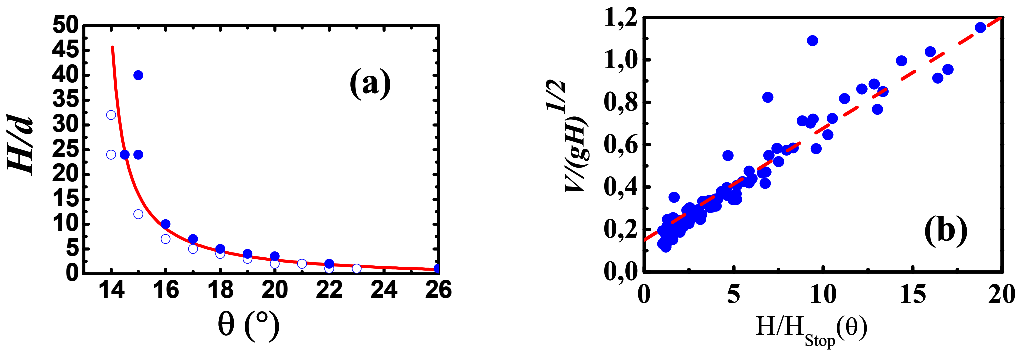

Figure 6 : Inclined plane (a) Jamming height (red curve). Empty and plain blue circles correspond to arrested or steady flow (b) Flow law. The four blue disks, which deviate from the simple law, correspond to thick flows (*H = 40*).

Jamming height

The first observation (Pouliquen, 1999) is that the jamming of a layer of grains flowing down a rough inclined plane depends not only on the inclination θ, but also on the height H. This defines a jamming height $H_{stop}(\theta)$, described by :

$$H_{stop}(\theta) = B\frac{\theta_M - \theta}{\theta - \theta_m} \qquad (3)$$

The parameters θ_m and θ_M (bounds for thick and thin layers) and B (length of influence of the roughness) depend on the material-roughness pair (da Cruz, 2004; GDR, 2004). In our two-dimensional simulations (Fig.6a), $\theta_m = 13{,}5°$, $\theta_M = 35°$ and $B/d \approx 1{,}2$.

Steady flow and macroscopic friction

The second observation (Pouliquen, 1999) is that, in the steady flows above the threshold (see the solid fraction and velocity profiles in Fig.7a), the dependence of the average velocity V as a function of H and θ ("flow law") is given by a simple relation between two dimensionless numbers, related to velocity and height, using the jamming height $H_{stop}(\theta)$:

$$\frac{V(H,\theta)}{\sqrt{gH}} = \beta \frac{H}{H_{stop}(\theta)} \quad (4)$$

with g the gravity and β a constant depending on the material-roughness pair ($\beta \approx 0.1$). The Fig.6b shows a measure of the flow law in our discrete numerical simulations.

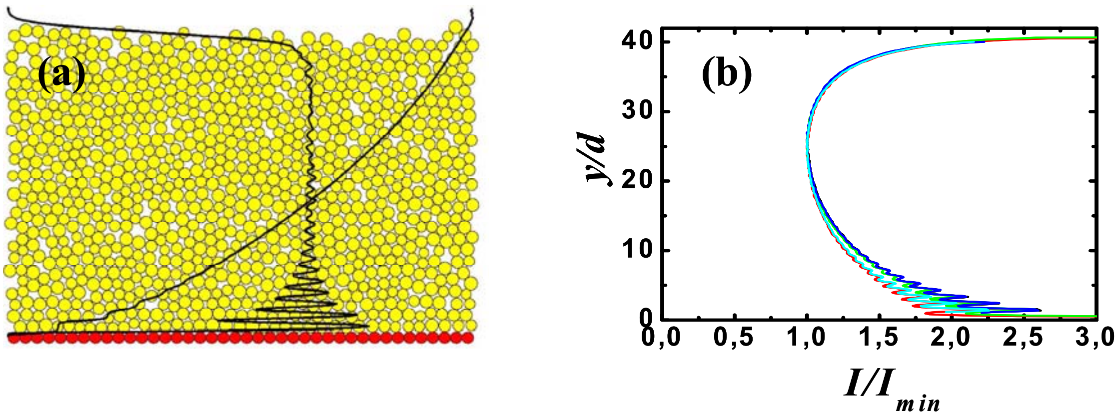

Figure 7 : Inclined plane (a) Picture of the flowing grains – Solid fraction and velocity profiles (H = 25, θ = 18°) (b) Normalised profiles of the inertial number for various inclinations (H = 40, θ between 15° and 21°)

From the two previous relations (3) and (4), it is possible to deduce the friction law, that is to say the dependence of the macroscopic friction on the inertial number. For a steady flow down an inclined plane, the macroscopic friction is equal to the inclination θ. The profiles of the inertial number (Fig.7b) show an increase of I near the free surface (possibly due to fluidisation) and near the bed (possibly due to the structuration in layers which slide more easily). The shape of those profiles does not depend on the inclination (Fig.7b), but the value at the center of the flowing layer I_{min} decreases near the jamming transition. Its variation with θ provides the friction law, shown in Fig.8a. Qualitatively similar to the one observed in homogeneous flows, the friction law down an inclined plane is shifted toward larger values, and, consistently with (3) and (4), may be adjusted by the following equation (with $\alpha = 2B/5\beta$) :

$$\mu^*(I) = \frac{\theta_m + \alpha\theta_M I}{1 + \alpha I} \quad (5)$$

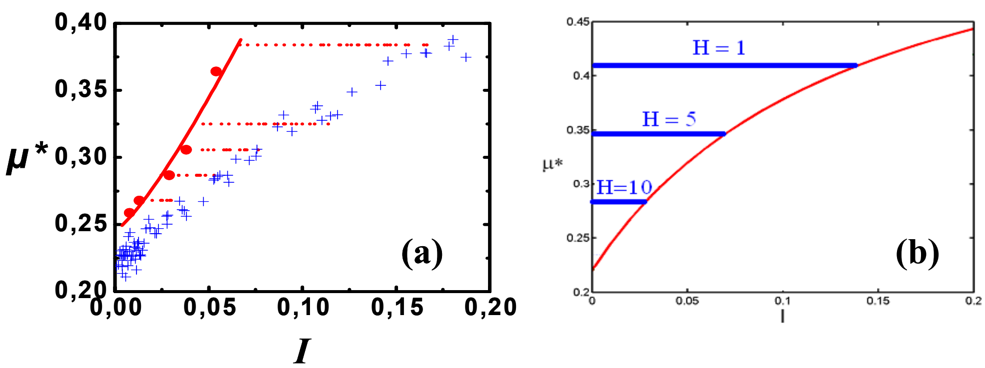

Figure 8 : Friction law down an inclined plane (a) Blue crosses : plane shear, red disks (CD), red curve (MD) : I_{min}, dotted red curves : I values spanned in the profiles (b) Trapping model – Effect of the flowing height on the jamming transition.

Jamming

According to the friction law (5), the flow should stop continuously when $\theta \to \theta_m$. In fact, one observes an abrupt arrest, when $\theta < \theta_{stop}(H)$ (da Cruz et al., 2002).This corresponds to a critival value of I depending on H ($I_{stop}(H) = 5\beta d/2H$). This rupture in the friction law is represented by the blue lines in Fig.8b. This sudden arrest is reminiscent of the observation of intermittencies and correlated motion of blocks in plane shear when $I \to 0$. When the flow slows down, the size of the blocks increases and when it becomes comparable to the height of the flowing layer, the flow stops. A phenomenological model has been proposed (da Cruz, 2004), based on the trapping of the grains initiated by the wall roughness, in good agreement with the previous observations (jamming height, flow law, friction law, dynamics of arrest) (Fig.8b). Starting from a steady flow and suddenly decreasing the inclination, it has also been possible to measure the evolution of the contact network near the jamming transition (Silbert *et al.*, 2002). The micromechanical quantities reveal a discontinuous transition from a value in the flowing state to a value in the solid state (Fig.9) : the mobilization of friction suddenly drops to zero, whereas the coordination number and the relative fluctuations of the translation velocity suddenly increase significantly. Consequently the jamming transition may be associated to a massive micromechanical transition.

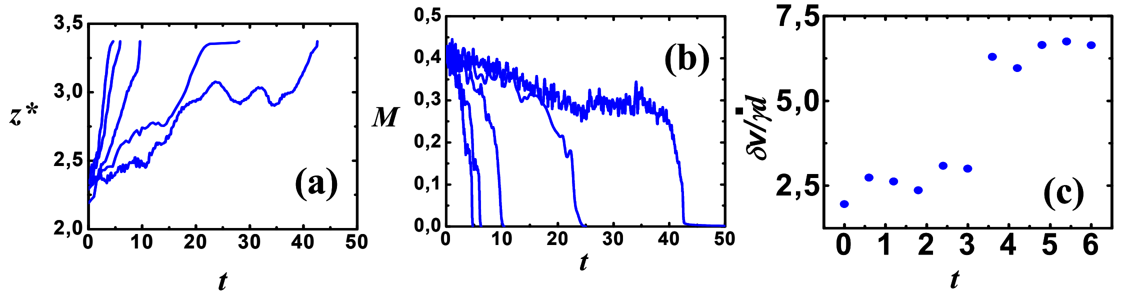

Figure 9 : Jamming transition – Time evolution after a sudden decrease of the inclination, starting from a steady flow (*H* = 20 - θ = 17°) (a) Coordination number (Δθ from 3° to 17°) (b) Mobilisation of friction (Δθ from 3° to 17°) (c) Relative fluctuations of the translation velocity Δθ = 12°).

CONCLUSION

We have shown that, in the limit of rigid grains, the constitutive law in the intermediate regime essentially depend on the dimensionless inertial number. The microscopic friction influences the maximum solid fraction, but not the macroscopic friction. Those conclusions have to be confirmed by studies in other geometries such as plane shear with gravity, annular shear cell, vertical channel or rotating drum (GDR, 2004). The influence of the polydispersity and of the shape of the grains should also be studied. We have also quoted two possible jamming mechanisms : correlated motions of the grains, and trapping initiated by the wall roughness. The complete understanding of those collective mechanisms remain an important challenge for the rheological models for granular materials.

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