Transient rigid clusters in dense granular flows

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Using discrete numerical simulations, we measure the spatial heterogeneity in dense granular flows, and identify transient rigid clusters immersed in an assembly of free grains. We measure the density and average size of the clusters as a function of the ratio of inertial to shear times, and show that they invade the granular flow in the quasi-static limit. Within this biphasic picture, the transmission of stress takes place through both the fluid phase and the rigid clusters, which induces a non local behavior. In the case of homogeneous shear, we deduce an equation of state and a friction law in agreement with discrete numerical simulations.

1 INTRODUCTION

Recent experimental and numerical studies have allowed the measurement of rheological laws in dense granular flows (Pouliquen and Chevoir 2002; GDR MIDI 2004; da Cruz et al. 2005). Several studies have revealed strong correlations of motion and force (Radjai and Roux 2002; Bonamy et al. 2002; Mueth 2003; Chambon et al. 2003; Ferguson et al. 2003; Pouliquen 2004). The observation of those dynamic heterogeneities have motivated the development of various non-local models (Pouliquen and Forterre 2001; Andreotti and Douady 2001; Chevoir et al. 2001; Lemaitre 2002; Ertas and Halsey 2002; Rajchenbach 2003; GDR MIDI 2004; Picard et al. 2005). Following a previous model (Mills et al. 1999; Bonamy and Mills 2003), we describe dense granular flows as transient rigid clusters immersed in a viscous fluid made of free grains interacting through collisions. Using molecular dynamics simulations described in Rognon et al. (2005), we identify candidates for those rigid structures, and measure their characteristics. Then we propose a rheological biphasic model with a non-local coupling term between the viscous fluid and the rigid clusters. At the end, we discuss the prediction of this model for jamming.

2 DENSE GRANULAR FLOWS

We consider two-dimensional steady homogeneous shear flows (plane shear without gravity) of an assembly of frictional inelastic rigid disks of diameter d and mass m, where both pressure P and shear rate $\dot{\gamma}$ are prescribed. Then, discrete numerical simulations (da Cruz et al. 2005) have shown that the flow regime is governed by the dimensionless number I, called inertial number, ratio of the inertial time $\tau_i = \sqrt{m/P}$ to the shear time $\tau_s = 1/\dot{\gamma}$:

$$I = \dot{\gamma}\sqrt{\frac{m}{P}}. \quad (1)$$

Large I correspond to the dynamic regime, where the grains interact through binary uncorrelated collisions, while small I correspond to the quasi-static regime, where the grains interact through a dense network of enduring contacts. In discrete simulations, approximately linear variations of the average solid fraction ν and of the effective friction coefficient μ^*, ratio of shear stress S to pressure P, are measured as a function of I :

$$\begin{cases} \nu \simeq \nu_m - aI, \\ \mu^* \simeq \mu^*_s + bI, \end{cases} \quad (2)$$

with $\nu_m \simeq 0.8$, $\mu_s^* \simeq 0.2$, $a \simeq 0.3$ and $b \simeq 1.0$ (for frictional grains). This rheological law in the intermediate regime is visco-plastic, and we now try to understand the contributions associated to collisions and to the enduring contacts. In contrast with previous biphasic approach based on the contact network (Radjai et al. 1998; Lemaitre 2002; Volfson et al. 2003), we shall focus on the spatial heterogeneity.

3 SPATIAL HETEROGENEITY

We first interpret the variation of the average solid fraction as the result of a balance between a compaction rate $(\nu_m - \nu)/\tau_i$ and a dilatancy rate $\alpha\nu/\dot{\gamma}$, so that $\nu = \nu_m/(1+\alpha I) \simeq \nu_m - \alpha\nu_m I$, in agreement with Eqn. (2).

However, we notice strong fluctuations of the solid fraction within the granular flow. As a way to characterize this spatial heterogeneity, we measure in discrete simulations (Rognon et al. 2005) the local solid fraction ν_i around a grain i as the ratio of the grain surface $\pi d^2/4$ to the surface of its Voronoï cell. The distribution of ν_i is shown in Fig.1 for various inertial number.

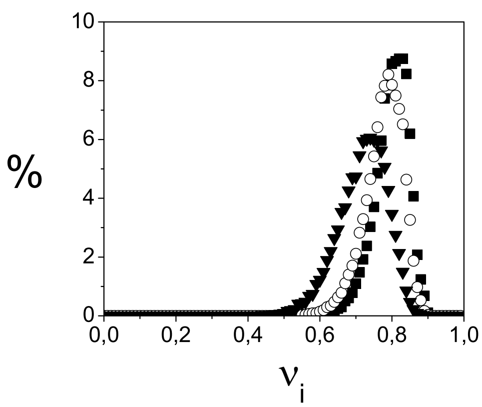

FIG. 1: Distribution of local solid fractions ($I = 0.025$ (■), $I = 0.1$ (○), $I = 0.3$ (▼).

Then we distinguish two populations of grains : those with $\nu_i \geq \nu_m$ and in contact with a neighbor grain constitute clusters, while the others are considered free. Pictures of these clusters are shown in Fig.2.

We call n the fraction of grains belonging to clusters, and $f \simeq n\nu/\nu_m$ the solid fraction of clusters. We consider that n is the result of a balance between a compaction rate $(1-n)/\tau_i$ and a dilatancy rate $\beta n\dot{\gamma}$, so that $n = 1/(1+\beta I)$. Consequently, we predict that :

$$f(I) \simeq \frac{1}{(1+\alpha I)(1+\beta I)}. \tag{3}$$

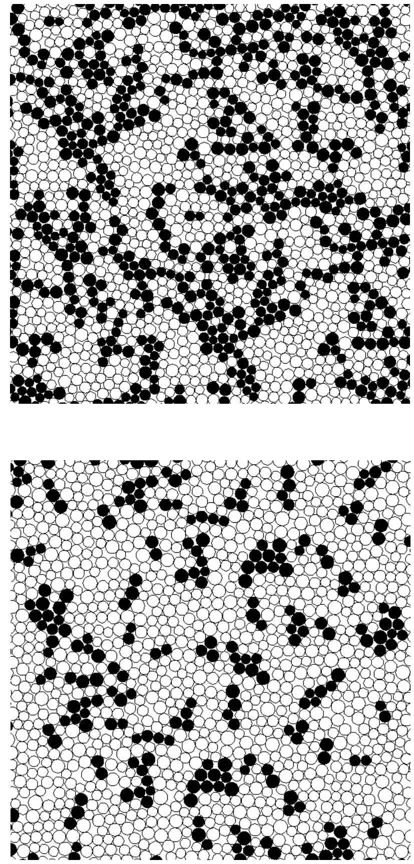

FIG. 2: Picture of the clusters : (a) $I = 0.025$, (b) $I = 0.1$.

The result of the measurement of $f(I)$ is shown in Fig.3a. $f(I)$ strongly decreases with I but does not tend to unity in the quasi-static limit. This indicates that a lower solid fraction threshold should be choosen to distinguish between free grains and clusters.

We observe a distribution of clusters of various size, characterized by their number of grains N_c ("mass") and gyration radius R_c. Fig.3b shows that the gyration radius follows the following scaling law as a function of mass :

$$R_c \sim N_c^D, \tag{4}$$

with an exponent $D \simeq 0.57$. From the "mass" distribution, we measure the following variation for the average mass (Fig.3c) :

$$\langle Nc \rangle \sim 1/I, \tag{5}$$

from which we deduce $\langle R_c \rangle \sim 1/I^D$. This means that the cluster size diverges in the quasi-static limit.

4 RHEOLOGICAL LAW

Within this biphasic approach, the shear stress is the sum of the contributions S^F of the free grains and S^C of the clusters, weighted by their solid fraction :

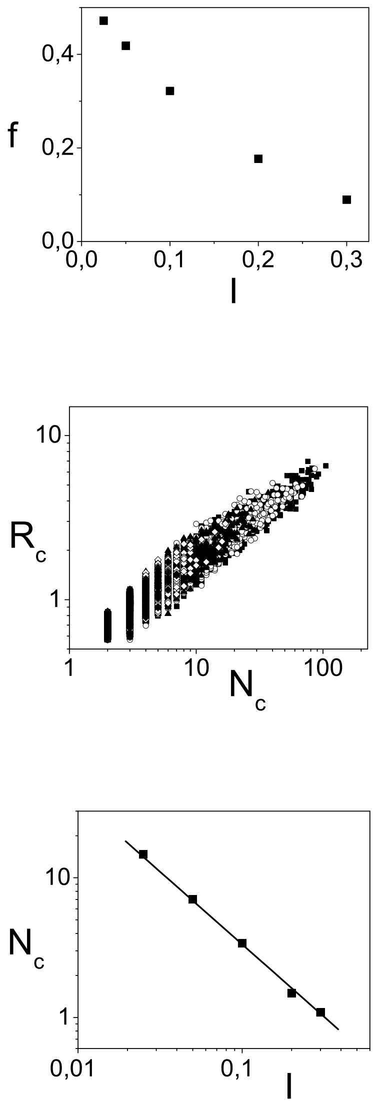

FIG. 3: Characterization of the clusters : (a) Solid fraction $f(I)$, (b) Scaling relation between mass N_c and size R_c ($I = 0.025$ - ■ ; $I = 0.05$ - ○ ; $I = 0.1$ - ▲ ; $I = 0.2$ - ◊ ; $I = 0.3$ - ◆) (c) Average mass $\langle Nc \rangle (I)$.

$$S = (1 - f)S^F + fS^C. \tag{6}$$

Our estimation of S^F is based on the usual argument in the collisional regime (Haff 1983) : the grains exchange a typical momentum $m\dot{\gamma}d$, with a frequency equal to their fluctuating velocity δv divided by the average (not smallest) distance between neighbor grain $s \simeq (1/\nu - 1)d$. In the dense regime, s tends to a non-zero value, so that :

$$S^F = A^F m(\delta v/d)\dot{\gamma}. \tag{7}$$

In the collisional regime, the fluctuating velocity is deduced from the energy equation ($\delta v \sim \dot{\gamma}d$), but discrete numerical simulations rather show $\delta v \sim I^{-1/2}d$ in the dense regime (da Cruz et al. 2005).

To evaluate S^C, our basic assumption is that the clusters are rigid structures through which the forces are transmitted. Then S^C is the sum of a quasi-static term $\tan\phi P$ inside the cluster (with the internal friction $\tan\phi$), and of a coupling term Σ between the clusters and the free grains. The clusters are transient structures, created by the sudden ”freezing” of a group of free grains. The lifetime of a cluster of size R_c is of the order of the propagation time of a solidification wave along the cluster ($(R_c/d)\tau_i$). Then Σ is given by the typical momentum transmitted through the cluster during its lifetime and over the one-dimensional cut of the cluster ($(R_c/d)^{D-1}d$). The transmitted momentum is the sum of the momentum $m\dot{\gamma}d$ of the $(R_c/d)^D$ grains which are suddenly frozen. Consequently, the coupling stress is evaluated as :

$$\Sigma = A^C(m/\tau_i)\dot{\gamma}. \tag{8}$$

This expression does not depend on the cluster size, so that it should hold for a population of clusters of various sizes.

The expression for the total shear stress S is consistent both with the Eqn. (7) in the collisional regime, when $f \to 0$, and with the friction law in the dense limit Eqn. (2) when $f \to 1$ (with $\mu_s^* = \tan\phi$ and $A^C = b$).

5 DISCUSSION

More work is needed to characterize those transient rigid clusters (choice of solid fraction threshold, measurement of the lifetime...).

The previous discussion concerned homogeneous shear flow. Then, the rheological law remains apparently ”local”, even if non-local transfer of momentum through the cluster takes place. But in the case of an heterogeneous shear distribution (plane shear with gravity, vertical chute, inclined plane flow...), the inertial number I and hence the characteristics of the clusters may vary along the shear gradient. A measurement of the clusters would be useful and a generalization of the previous model must be written. For steady flows down a rough inclined plane, the inertial number is aproximately constant within the flowing layer (da Cruz et al. 2005) (except near the rough wall), so that the previous apparently local model applies. However, as the inclination θ decreases, the average cluster size increases and when it reaches the height H of the flowing layer, non-local mechanisms should have a stronger influence (GDR MIDI 2004). This defines a curve $H_{nl}(\theta) \sim (\theta - \phi)^{-D}$ reminiscent of the $H_{stop}(\theta)$ curve (Pouliquen 2004). The jamming mechanism should be described properly and should take into account the interaction of the clusters with the rough wall.

In the case of a confined flow, such as plane shear flow between two walls distant of H, when the average cluster size becomes comparable to H, continuous steady flow is no more possible. A transition should occur for a critical inertial number I_c, depending on H : jamming if the shear stress is prescribed, and intermittencies if the shear rate is prescribed. This is indeed observed in discrete simulation of confined plane shear flow (da Cruz et al. 2005).

We think that this kind of model may apply to other concentrated particulate systems (colloidal suspensions, emulsions, foams...) (Farr et al. 1997; Cates et al. 1998; da Cruz et al. 2002; Picard et al. 2005).

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