Dense flows of bidisperse assemblies of disks down an inclined plane

P. G. Rognon, J.-N. Roux & F. Chevoir
LMSGC, UMR LCPC-ENPC-CNRS, Institut Navier 2 allée Kepler, 77 420 Champs sur Marne, France

M. Naaïm
CEMAGREF, 2 rue de la Papeterie, BP 76, 38402 Saint-Martin d'Hères, France

Using molecular dynamic simulations, we have studied the flow down a rough inclined plane of a bidisperse assembly of frictional cohesionless disks, characterized by the size ratio and the proportion of large grains. We have restricted our study to steady uniform flows, once a stable segregation has developed inside the flowing layer. The material is segregated in three layers : a basal layer made of small grains, a superficial layer made of large grains, and a mixed layer in the center. In a certain range of parameters of the mixture, this structure leads to a localization of the shear near the rough bed. Moreover, like for slightly polydisperse assemblies, we measure an approximately linear increase of the effective friction as a function of the ratio of the Froude number and the height of the flowing layer. We compare those results with the prediction of a simple two-layer model.

1 INTRODUCTION

Recent experimental and numerical studies have improved the understanding of quasi-monodisperse granular flows down rough slopes (Pouliquen & Chevoir 2002; GDR MIDI 2004; da Cruz et al. 2005). The friction law has been deduced from the measurement of the velocity as a function of inclination and height in steady flows (Pouliquen & Chevoir 2002). The use of this friction law within Saint-Venant approach makes it possible to simulate large scale granular flows (Pouliquen & Forterre 2002; Naaim et al. 2004; Mangeney et al. 2003).

But natural gravity flows - snow avalanches (Bouchet et al. 2003), rock falls, pyroclastic flows (Felix & Thomas 2004) - are usually made of grains of various sizes. This polydispersity leads to segregation effect during the flows (Hirshfeld & Rapaport 1997; Berton et al. 2003), but the consequences on the rheological laws are not very well known yet (Goujon 2004). This has motivated our discrete numerical study (using molecular dynamics method) of the simple case of bidisperse granular flows down a rough inclined plane. We shall first identify the parameters describing the simulated system, and then show how the segregation affects the velocity profiles and the friction law.

2 SIMULATED SYSTEM

The system is two-dimensional. The granular material is a bi-disperse assembly of 500 to 1000 disks of the same mass density ρ_p: n_s small disks of average diameter d_s and n_l large disks of average diameter d_l (with a small polydispersity ($\pm 20\%$) around each average diameter). This mixture is characterized by the size ratio $D_r = d_l/d_s$ and the areal proportion of large grains $S_r = (n_g {d_g}^2)/(n_g {d_g}^2 + n_s {d_s}^2)$. We have studied the following mixtures : $D_r = \{2;3;4;6;8\}$ and $S_r = \{1/4;1/2;3/4\}$.

The flows are simulated by standard molecular dynamics method adapted to visco-elastic frictional grains (Roux & Chevoir 2005). The simulations were done with a friction coefficient 0.4, a coefficient of normal restitution 0.1, and in the limit of rigid grains (the relative deformation of the grains remain smaller than 10^{-3}). Consequently, the gravity time scale $\sqrt{d_s/g}$ is much larger than the collision time.

A dense layer of height H flows down a rough inclined plane (inclination θ) under the influence of gravity g (Fig. 1). Periodic boundary conditions are applied along the flow direction x. The roughness is made of contiguous grains sharing the characteristics of the small flowing grains.

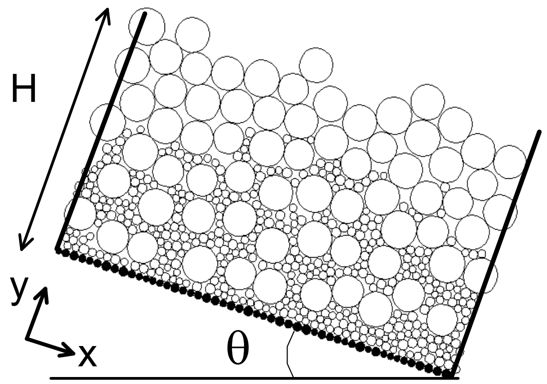

Figure 1. Bidisperse granular flow down a rough inclined plane ($D_r = 4, S_r = 3/4, \theta = 17^o, H = 28d_s$).

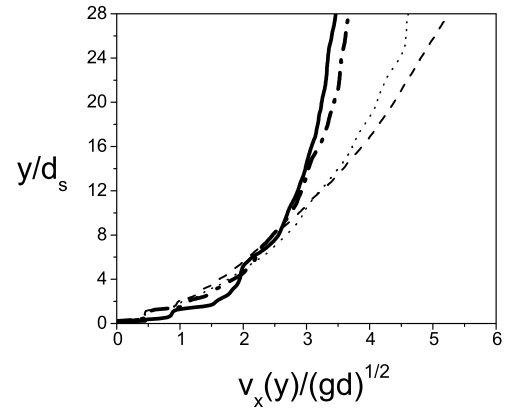

Figure 2. Velocity profiles ($\theta = 17^o, H = 28d_s$, $D_r = 4$) $S_r = 0$ (- -), 1/4 (...), 1/2 ($- \cdot -$), $3/4$ (—).

3 STEADY FLOWS

In the following, we focus on the steady flows which are possible in a range $(\theta; H)$ which depends on the mixture. Averaging over time and space (along x), we measure the profiles of solid fraction, velocity and stress tensor along the transverse direction y.

3.1 Structure

The solid fraction profiles $\nu(y)$ are approximately constant, with an average solid fraction ν around 0.8 (which very slightly depends on the composition of the mixture). The grains are organized in three layers (Fig. 1): a layer of small grains near the rough bed, a layer of large grains near the free surface and a mixed layer at the center. This segregated state is stable in time and independent of the initial arrangement of the grains. We observe that the normal stress components are equal. Consistently with momentum conservation, the pressure verify $P(y) = \rho_p \nu g \cos\theta(H - y)$ and the shear stress $S(y) = \rho_p \nu g \sin\theta(H - y)$.

3.2 Velocity profiles

Fig. 2 shows the influence of the polydispersity on the velocity profile $v_x(y)$. If the granular material is quasi-monodisperse, the velocity profile has a Bagnold-type shape (Silbert et al. 2001; Prochnow 2002), which means that the shear rate $\dot{\gamma}(y)$ is proportional to $\sqrt{P(y)}$. We systematically observe a decrease of the shear rate in the upper part of the flow as S_r or D_r increases. Moreover, for a large enough proportion of large grains ($S_r \geq 3/4$) and for a large enough size ratio ($D_r \geq 3$), we also observe a strong increase of the shear rate near the bed.

3.3 Discussion

We now try to relate this localization of the shear to the segregation of the granular material. The study of steady homogeneous shear flows of assemblies of quasi-monodisperse grains of average diameter d, with prescribed pressure P and shear rate $\dot{\gamma}$, has shown that the shear state is controlled by a single dimensionless number, called inertial number, $I = \dot{\gamma} d\sqrt{\rho_p/P}$ (da Cruz et al. 2005). The effective friction coefficient $\mu^* = S/P$ increases approximately linearly with I, starting from the internal friction in the critical state $\tan\phi \simeq \phi$:

$$\mu^* \simeq \phi + bI. \tag{1}$$

We use this "friction law" to discuss the profile of shear rate for a steady dense flow down a rough inclined plane. Then μ^* is equal to $\tan\theta \simeq \theta$, and this gives rise to the Bagnold scaling :

$$\dot{\gamma}(y) \simeq \frac{\sqrt{\nu g}(\theta - \phi)}{bd}\sqrt{H - y} \tag{2}$$

Then, for given inclination θ and height H, the shear rate should decrease as the inverse of the diameter of the grains. The ratio Γ of shear rates in two layers made of small or large grains should be equal to the inverse of the size ratio :

$$\Gamma = \frac{\dot{\gamma}_l}{\dot{\gamma}_s} = \frac{d_s}{d_l} = D_r^{-1} \tag{3}$$

Let us now consider a simplified two-layer picture of bidisperse flows, where the segregation is complete: the flow is made of a layer of small grains near the bed (width $(1 - S_r)H$), and a layer of large grains near the free surface (width $S_r H$). Then, in comparison with a monodisperse flow of small grains ($S_r = 0$), the ratio Γ should be equal to 1 in the lower layer and to $1/D_r$ in the upper layer. This prediction for $\Gamma(y)$ is compared with the measurement in Fig. 3 for $D_r = 4$ and two values of S_r ($1/2$ and $3/4$). For $S_r = 1/2$, the model is correct for the upper and lower layers ($y \geq 0.75H$ and $y \leq 0.1H$) made of one type of grains. But the mixed layer at the centre is not well represented. For $S_r = 3/4$, the model is correct for the upper layer ($y \geq 0.75H$) made of large grains only, but the increase of the shear rate near the roughness is not described. We think that this sliding is the consequence of a less efficient trapping of the large grains by the roughness (Goujon et al. 2003). This sliding should increase with the size ratio and the proportion of large grains.

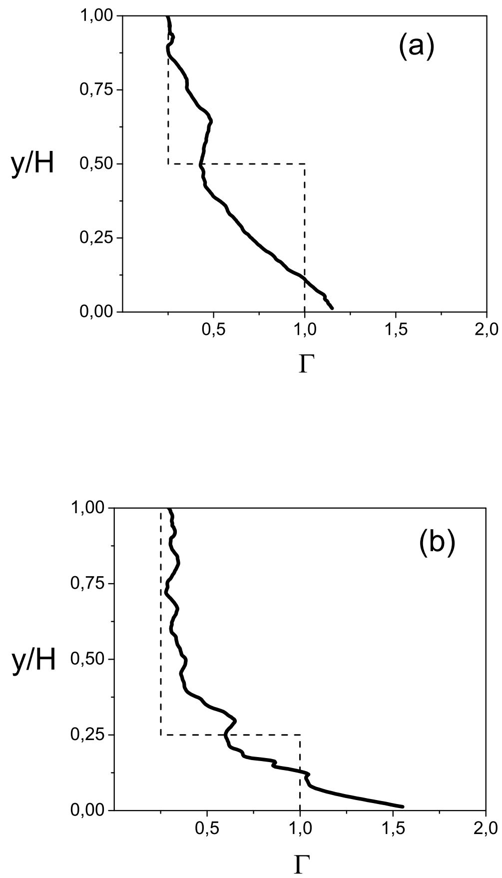

Figure 3. Comparison of shear rates ($\theta = 17^o,\ H = 28d_s,\ D_r = 4$). (a) $S_r = 1/2$, (b) $S_r = 3/4$. Measurement (line), model(dot).

4 FRICTION LAW

For the flow of a bidisperse granular mixture down a rough inclined plane, the local inertial number previously defined may vary within the flowing layer. Then we rather define a global inertial number, from the average shear rate $v_x(H)/H$ and the pressure at the bed $P(0)$ (da Cruz et al. 2005) :

$$I_g = \frac{v_x(H)}{H} d \sqrt{\frac{\rho_p}{P(0)}}. \tag{4}$$

This is equivalent to the parameter $Fr/(H/d)$ introduced by Pouliquen (1999), where $Fr = v_x(H)/\sqrt{gH}$ is the Froude number. In the case of a monodisperse granular flow, the assumption of a constant I profile (see however (da Cruz et al. 2005)), leads to $I_g = 2I/3$. Then, introducing $B = 3b/2$, the friction law is :

$$\mu^* \simeq \phi + BI_g. \tag{5}$$

4.1 Measurements

For each bidispere mixture, we change the inclination θ of the rough plane slowly enough that the flow can be considered steady at each time step. Then we measure, as a function of $\theta \simeq \mu^*$, the height H and the surface velocity $v_x(H)$, from which we calculate I_g. This gives access to the friction law of the bidisperse mixture. The figure 4 shows that the curves measured for various H collapse for $H \geq 10d_s$. The friction law is similar to Eqn. (5) for monodisperse granular flows. Contrarily to the slope B, the internal friction angle ϕ depends on the composition of the bidisperse mixture, as shown on Fig. 5. For $S_r \leq 1/4$, the size ratio has no influence. But for $S_r \geq 1/2$, ϕ decreases when D_r increases.

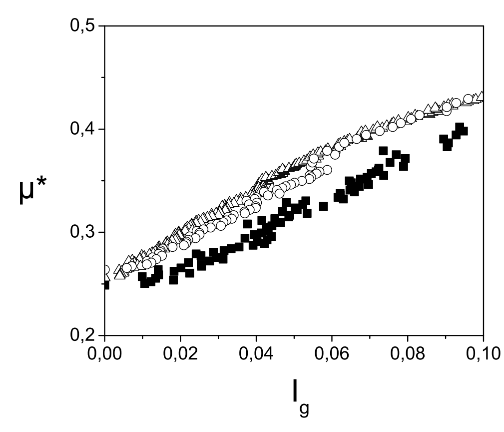

Figure 4. Friction law ($D_r = 3,\ S_r = 1/2$), $H/d_s = 10\ (\blacksquare),\ 20\ (\triangle),\ 30\ (\circ)$.

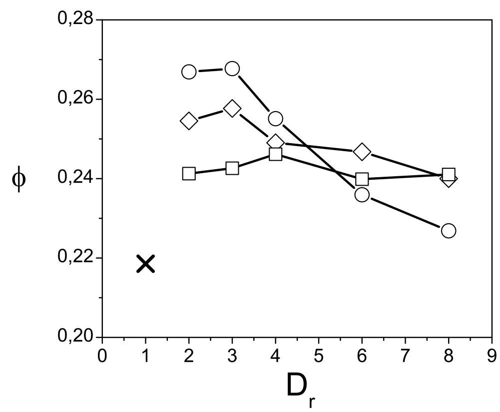

Figure 5. $\phi(D_r; S_r)$ $S_r = 0\,(\times), S_r = 1/4(\Box),\ 1/2(\Diamond),\ 3/4(\circ)$.

4.2 Discussion

The previous two-layer model predicts the same linear increase of the effective friction as a function of the global inertial number:

$$\mu^* \simeq \phi + B(S_r, D_r) I_g, \tag{6}$$

with a strong influence of the composition of the bidisperse mixture on the slope :

$$B(S_r, D_r) = \frac{B}{(1-S_r)^{3/2} + S_r^{3/2}/D_r}, \tag{7}$$

but no influence on the internal friction ϕ, contrarily to the measurement. This reveals that the rheology of the mixed layer and, most of all, the interaction of the mixture with the roughness (Goujon et al. 2003) have a strong influence on the global rheology.

5 CONCLUSION

Using molecular dynamics simulations, we have identified the influence of large grains on bidisperse granular flows down a rough inclined plane. The material is segregated in three layers : a basal layer made of small grains, a superficial layer made of large grains, and a mixed layer in the center. Large grains decrease the shear rate in the upper layer but favors sliding at the bed. The dependency of the friction coefficient as a function of the global inertial number has been measured. The internal friction depends on the composition of the bidisperse mixture. We have integrated the local rheology in a simple two layer model where the material is completely segregated. The model explain the decrease of the shear rate in the upper layer, and predicts qualitatively the friction law. But discrepencies with the measurements indicate the strong interaction of the bidisperse flow with the small grains roughness.

REFERENCES

Berton, G., Delannay, R., Richard, P., Taberlet, N., & Valance, A. 2003. Two-dimensional inclined chute flows : Transverse motion and segregation. *Physical Review E* 68: 051303.

Bouchet, A., Naaim, M., Ousset, F., Bellot, H., & Cauvard, D. 2003. Experimental determination of constitutive equations for dense and dry avalanches : presentation of the set-up and first results. *Surveys in Geophysics* 24: 525–541.

da Cruz, F., Prochnow, M., Roux, J.-N., & Chevoir, F. 2005. Dense granular flows: Friction and jamming. In *in these proceedings*.

Felix, G. & Thomas, N. 2004, April. Relation between dry granular flow regimes and morphology of deposits : formation of leves in pyroclastics deposits. *Earth and Planetary Science Letter* 221: 197–213.

GDR MIDI 2004. On dense granular flows. *Euro. Phys. J. E* 14: 341–365.

Goujon, C. 2004. *Bidisperse granular flows down rough inclined planes*. Ph. D. thesis: Université de Provence. in French.

Goujon, C., Thomas, N., & Dalloz-Dubrujeaud, B. 2003. Monodisperse dry grain flows on inclined planes : role of roughness. *Euro. Phys. J. E* 11: 147–157.

Hirshfeld, D. & Rapaport, D. 1997. Molecular dynamics studies of grain segregation in sheared flow. *Physical Review E 56*(2): 2012–2018.

Mangeney, A., Vilotte, J.-P., Bristeau, M.-O., Perthame, B., Simeoni, C., & Yerneni, S. 2003. Numerical modelling of avalanches based on Saint Venant equations using a kinetic scheme. *J. Geophys. Res.*. in press.

Naaim, M., F., N.-B., Faug, T., & Bouchet, A. 2004, October. Dense snow avalanche modeling : flow, erosion, deposition and obstacle effets. *Cold Regions Science and Technology 39*(2-3): 193–204.

Pouliquen, O. 1999. Scaling laws in granular flows down a rough inclined plane. *Phys. Fluids* 11: 542–548.

Pouliquen, O. & Chevoir, F. 2002. Dense flows of dry granular materials. *Compte Rendu Physique* 3: 163–175.

Pouliquen, O. & Forterre, Y. 2002. Friction law for dense granular flow : application to the motion of a mass down a rough inclined plane. *J. Fluid Mech.* 453: 133–151.

Prochnow, M. 2002. *Dense flows of dry grains*. Ph. D. thesis: Ecole Nationale des Ponts et Chaussées. in French.

Roux, J. N. & Chevoir, F. 2005. Mechanical behaviour of granular materials and discrete numerical simulations. *Bulletin des Laboratoires des Ponts et Chaussées*. to be published.

Silbert, S. L., Ertas, D., Grest, G. S., Halsey, T., Levine, D., & Plimpton, S. J. 2001. Granular flow down an inclined plane. *Phys. Rev. E* 64: 385–403.