Rheology of dense snow flows
Inferences from steady state chute-flow experiments

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Abstract

Good knowledge of snow rheology is useful for the mitigation of avalanches. However, experiments with snow are difficult and the few available data provide only a partial knowledge of snow flows. In this study we investigated the rheological behavior of a dense flow of dry snow, which often occurs in real avalanches. To this end, we carried out systematic small-scale in-situ flows down a flume with natural snow. Over three winters, we performed approximately 100 experiments with various slopes and flow discharges and we characterized them by measuring the velocity profile and basal stress. This data set, unique in its extent, allows us to identify various generic characteristics of dense flow of dry snow, which are found to differ from common fluids. We point out that snow flows develop as a very viscous upper thick layer over a much less viscous thin layer. We interpret this heterogeneity as a consequence of a shear-induced evolution of the snow microstructure that gives rise to different materials between the lower part made of single snow grains and the upper layer made of large aggregates. Finally, we show that a single constitutive law that describes dense flow of cohesionless grains can represent the behavior of each layer assuming different grain sizes. Beside its practical importance, this study on snow flow provides new insights into the rheological behavior of similar materials: the wide variety of cohesive granular materials such as humid sand, powders, and bituminous suspensions.

1 Introduction

Snow avalanches cause extensive economic damage in mountainous areas. Understanding snow flow is crucial for risk mitigation, via a better prediction of hazard zones and the optimization of costly defense structures such as dams and braking mounds. Despite these practical needs, the characteristics of the snow flow are still largely unknown. This is partially the result of the problems inherent to experimenting with snow, but the major obstacle is the complexity of the material: not only does snow belong to the wide variety of cohesive granular materials such as humid sand and powders, but its microstructure also evolves over time as function of the thermodynamical conditions. As a consequence, snow grains may exhibit various shapes, sizes and cohesive interactions. Gaining new insight into snow flow rheology therefore requires identification of the interplay between the flow properties and the grain properties, especially their cohesive interaction.

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In this paper we investigate the rheological behavior of dense flows of dry snow. To this end, we carried out systematic small-scale in-situ flows down a flume with natural snow over three winters. Experiments at high altitude are unusual in their logistical complexity, but are necessary to have access to natural snow. We performed approximately 100 runs with various slopes and flow discharges, and we characterized them by measuring the velocity profile and basal stress. From this extensive database, we identify various generic characteristics of the dense flow of dry snow that are found to differ from common fluids. We interpret this behavior as a consequence of a shear-induced evolution of the snow microstructure, and show its similarities with the behavior of dense granular flows.

The introduction presents various aspects of snow morphology, and reviews the previous experimental studies on dense flow of dry snow. The experimental procedure is detailed in § 2. Snow flow behavior is identified first from general observations such as steady and uniform flows and the discharge equation in § 3, then from the analysis of the velocity profile in § 4. An interpretation of this behavior is proposed in § 5. Conclusions and future perspectives are drawn in § 6. Preliminary results can be found in Bouchet *et al.* (2003); Bouchet (2003); Bouchet *et al.* (2004); Rognon (2006); Rastello & Bouchet (2007).

1.1   Different kinds of snow

Snow is a particuliar granular material since the size of the snow grains, their shape and their mechanical properties continuously evolve over time as a function of thermodynamical conditions (Marbouty, 1980; Colbeck, 1983; Brown *et al.*, 2001). This process, referred to as snow metamorphism, gives rise to various kinds of snow grains within the snow cover. They can be roughly classified into three categories. *Fresh snow* corresponds to ice crystals like those formed in clouds, with ice branches a few millimeters long, called dendrites, whose tangle gives rise to *felting cohesion* (Nakaya, 1954; Reiter, 2005). However, since the dendrites are weak and easily sublime, fresh snow tends to transform into smaller (0.1 to 0.5 mm) and more spherical grains, referred to as *fine grains*. Then felting cohesion vanishes while sintering forms small ice bonds between grains (Kuroiwa, 1974; Colbeck, 1998). This grain type constitutes the main mass of snow cover at freezing temperatures. At positive temperatures both ice bonds and grain surfaces melt, and fine grains transform into larger grains (a few millimeters) linked via the *capillary cohesion* of liquid menisci.

A great deal of research has focused on the relationship between the macroscopic mechanical behavior of the snow cover and the properties of snow grains (Voitkovsky *et al.*, 1974; St Lawrence & Bradley, 1974; Kry, 1975; Gaméda *et al.*, 1996; Johnson & Schneebeli, 1999; Schweizer & Camponovo, 2001; Lehning *et al.*, 2002; Haenel & Shoop, 2004). In contrast, little is known about the interactions of snow grains within a flow. The studies of binary collision between two artificial ice spheres measuring a few centimeters pointed out the dependence of energy dissipation and adhesion on temperature and the existence of a thin liquid layer at the surface of grains (Bridges *et al.*, 1984; Hatzes *et al.*, 1988, 1991; Higa *et al.*, 1995; Supulver *et al.*, 1995). However, these large ice spheres differ substancially from natural snow grains, which are generally much smaller and porous.

1.2   Experimental knowledge on dense snow rheology

Snow flow properties were investigated using two complementary approaches: artificial triggering and small-scale experiments. Several full-scale avalanche test sites provided interesting information on velocity and wall stresses of snow avalanches, and are therefore useful for engineering purposes (Naaim & Naaim-Bouvet, 2001; Dent & Lang, 1983; Qiu *et al.*, 1997; Dent *et al.*, 1998; Vallet *et al.*, 2001; Meunier *et al.*, 2004). Such studies also pointed out the distinction between two types of snow flows, namely *powder flows* and *dense flows*. Powder flows are made of snow grains fluidized in air (with a very low density $\sim$ 1-10 kg/m^3), that move very fast more and less independently of the relief. In contrast, dense flows (density between 100 and 500 kg/m^3) follow the slope and are made of a continuous network of contacting grains. Natural snow avalanches usually contain a basal dense flow above which a powder flow develops.

Although they give access to crucial information, full-scale experiments are not controlled and impossible to reproduce, and measurements within flows are difficult to obtain.

Small-scale experiments can be much more easily controlled and are therefore more appropriate for a rheological investigation. The typical approach consists in releasing a dense snow flow down an inclined channel, either set in a cold room (Nishimura & Maeno, 1989) or outdoors at high altitude (Tiefenbacher & Kern, 2004; Kern *et al.*, 2004; Bouchet *et al.*, 2003, 2004), and then analyzing internal velocity profiles in rheological terms. Despite the major differences in experimental procedures (channel size, snow preparation, etc.), the velocity profiles were found to be remarkably consistent with the velocity profile measured on full-scale avalanches by Gubler (1987); Dent *et al.* (1998): snow flows are highly sheared in a thin basal layer and much less sheared in the upper thick part. This velocity profile suggests the behavior of a yield stress fluid for which the free surface flow over an inclined plane exhibits a plugged region above some critical depth. In this context, the Bingham (Nishimura & Maeno, 1989), Herschel-Bulkley and Cross models (Kern *et al.*, 2004), and the biviscous model (Dent & Lang, 1983) were used to represent snow behavior. These models provided satisfactory fits of snow velocity profiles when adjusting a non-null slip velocity at the bottom of the flow. Thus, the existing experimental data suggest that the snow behavior may be described by simple constitutive laws. However, this conclusion is rather tentative since it is based on only a few experiments, and it was not checked whether one of these models is able to predict the flow characteristics of snow for different slopes and flow rates.

Since snow is made up of grains, several studies indirectly investigated snow avalanche behavior through extensive experiments with granular materials such as glass beads. Friction laws that describe granular flow down an inclined plane (Savage, 1979; Savage & Hutter, 1989; Pouliquen, 1999; Louge & Keast, 2001; GdR MiDi, 2004) are often used in full-scale avalanche simulation using the Saint-Venant approach (Naaim *et al.*, 1997; Mangeney-Castelnau *et al.*, 2005). Interaction between granular flow and obstacles provided scaling laws allowing the design of efficient defense structures to protect against snow avalanches (see for example Faug *et al.* (2007) and references therein). Rapid granular chute-flow experiments pointed out the formation of a dilute layer at the free surface of dense flow, which may favor the fluidization of small grains such as snow grains (Barbolini *et al.*, 2005*b*), and also investigated the strong interaction between the flowing material and an erodible bed (Barbolini *et al.*, 2005*a*), a crucial process for snow avalanches (Naaim *et al.*, 2004). However, to what extent granular flows are similar to snow flows is still an open question, since the comparison would require many more experiments with natural snow than what exists so far. While granular experiments generally involve cohesionless grains, one can expect that intergranular cohesive forces between snow grains play an important role in the rheological properties. And since this cohesion significantly evolves throughout snow metamorphism, it is not clear whether a generic behavior exists for dry snows.

2 Experimental procedure

The snow flows have been performed over 3 years at the experimental in-situ test site situated at the *Col du Lac Blanc*, a pass near the Alpe d'Huez ski resort in the French Alps. The high altitude (2830 m a.s.l.) provides access to large amounts of natural snow between January and April. The set-up and instrumental devices were described in Bouchet *et al.* (2003); Bouchet (2003); Bouchet *et al.* (2004); Rognon (2006).

2.1 Flow geometry and feeding system

The flow geometry is a 10 m long channel (Fig.1 a). Its width and height are 20 cm. The slope can be set from 27° to 45°. To prevent slip, the channel bottom was covered with sand paper with a roughness of the order of the snow grain size (~ 0.4 mm). In contrast the lateral walls were smooth (PVC) so that the material could easily slip. Our measurements showed that the velocity at the free surface of the flow was almost constant in a cross-section (see Section 4.1), which suggests that the lateral walls affected the

flow characteristics only slightly, probably due to wall slip. Actually, Jop *et al.* (2005) showed that even in that case channelized granular flows can be affected by the lateral walls, but taking into account this effect here would not change the conclusions below. Under these conditions, we assume that the flow characteristics are those of the flow over an infinitely wide plane, and in steady-state so that the stress distribution can be deduced from the momentum equation (Coussot, 2005).

The feeding system is an important feature of the experimental set-up. It is made up of a hopper, which can store up to 5 m^3 of snow, and an Archimedean screw 4 m long and 0.6 m in diameter. The screw injects the snow into the channel at a constant flow rate that can be adjusted up to 0.1 m^3/s by varying the rotational frequency (up to 1 Hz). Although the flow rate averaged over a period of rotation is constant, it depends slightly on the orientation of the screw, leading to periodical variations on the order of 20%. In order to limit the effect of such variations on the flow characteristics, at the beginning of the chute we set up a system that deviates the upper part of the flow outside the flume and thus ensures a constant flow rate downstream.

Figure 1: Experimental set-up at the *Col du Lac Blanc* (Alpes d'Huez, 2830m). (a) Set-up: (1) moving tank, (2) hopper, (3) Archimedean screw, (4) channel. (b) Position of sensors (flow depth $H_{1,2,3}$, velocity profile V and basal stresses P and τ): lateral view of the chute, and schematic views (c) from the top, and (d) from the side.

2.2 Measurements

The sensors are set around the middle of the chute (Fig. 1 b-d). They record the flow height H at three points along the flow direction, the normal (P) and shear (τ) stresses at the bottom and the velocity profile $V_x(z)$ along the depth z. The data are recorded along the entire duration of flows with a sampling

frequency $\mathcal{F}_{acq}$=10 kHz. The three height sensors (LEUZE ODS M/V-5010-600-421) are fixed above the channel (z=0.3 m) at y=0. They emit visible light and detect the position of the beam reflected by the flow. They provide the flow thickness from 0 to 20 cm with a precision of 1 mm. The stress sensor is a bi-component piezo-electric (Kistler $9601A21-2-20$) sandwiched between two metal plates 0.2 m (along the channel width) by 0.5 m. It is inserted into the bottom of the channel and its upper surface is covered with sand paper.

The velocity measurements are based on the correlation of two signals from identical sensors located at two successive positions along the channel (Dent *et al.*, 1998; Bouchet *et al.*, 2003). Each sensor is an optical device that emits infrared light and measures the intensity of the reflected beam. The corresponding signal is related in a non-trivial way to the density, granularity and microstructure of the reflecting snow. Two such devices located at a distance δ=7.2 mm along the flow direction give similar signals $X(t_i)$ and $Y(t_i)$, but shifted by a time $\Delta t(z) = \delta/V_x(z)$ where $V_x(z)$ is the velocity (Fig.2 a). The value of $\Delta t_i(z)$ is the time shift for which the discrete cross-correlation function $\mathcal{C}(z)$ between the signals $X(t_i, z)$ and $Y(t_i, z)$ reaches its maximum (Fig.2 b). The similarity of the two signals $X(t_i, z)$ and $Y(t_i, z)$ is good if the microstructure of the moving snow is constant, so that the distance δ must be as small as possible. As a counterpart, the shift time is small ($\Delta t_i \approx 1.5$ ms for a typical velocity of 5 $\mathrm{m\,s^{-1}}$), and its relative precision $1/(\Delta t_i \times \mathcal{F}_{acq}) \approx 7\%$ is low. To obtain a better precision, the discrete correlation function is fitted with a quadratic function including ten points around Δt_i, and Δt is defined as the time shift that peaks this function (Fig.2 b, insert). The apparatus for the velocity profile consists of 13 pairs of sensors stacked along z (Fig.2 c). Then the column is inserted into a lateral wall, setting the bottom of the first sensor at z=0. The sensors are dispatched every centimeter, except for the first two cm where the spatial resolution is increased (0.5 cm), which requires placing the sensors in two columns.

Figure 2: Velocity measurements: (a) typical signals recorded from a pair of optical sensors, (b) cross-correlation $\mathcal{C}(\Delta t_i)$ of these signals, inset: quadratic fit around the peaks of $\mathcal{C}(\Delta t_i)$, (c) distribution of sensors.

2.3 Procedure

Fifteen series of experiments were performed over three winters, providing 85 flows (Fig. 3 a). For each series 2 days were needed to clean the channel, which was often buried under several meters of snow, install the electronic devices, and repair the damage to the mechanical structure caused by the harsh

weather. The experiments were performed at night at temperatures ranging from -25°C to -3°C, which prevents the presence of liquid water. Snow cover was collected nearby and was prepared as a mixture of individual grains (typical diameter ranging from 0.1 to 0.5 mm) or clusters smaller than 2 cm by passing it through a grinder. This mixture was then stored in the moving tank. During this operation (which lasted approximately 20 minutes) the ice grains could sinter and form a solid block inside the moving tank. A few minutes before the beginning of the experiments, the moving tank was emptied into the hopper through a 3 cm-mesh sieve, which broke the large snow blocks into a mixture of individual grains and aggregates of a diameter smaller than a mesh width. Then the Archimedean screw was activated and the mixture was injected into the channel. Up to ten runs per night could be performed.

Figure 3: *Details of performed runs: (a) number of runs for each series (the labels inside are year), (b) ranges of slope θ and flow depth H for which steady uniform flows were obtained.*

3   General properties of flow

We first present the overall characteristics of the snow flows, from the evolution of the mean velocity over the depth, $U = \frac{1}{H} \int_{z=0}^{H} V_x(z)dz$, as a function of the slope and flow discharge.

3.1   Flow regimes

Since the flow rate (Q) imposed by the feeding system is constant during the flow, and neglecting variations of the snow density along the flume axis, the mean velocity (U) in the direction x is readily found from the mass conservation: $U(x) = Q/H(x)$, in which $H(x)$ is the flow height. From the measurements of the flow thickness along the channel axis, we observed three flow regimes depending on the flume's slope angle θ (Figure 4). For a slope lower than approximately 33°, the flow depth increases downstream, which means that the average velocity U decreases downstream. This corresponds to a *decelerated flow regime*. When the flow decelerates enough, it stops inside the channel and the snow forms a rigid block within a few seconds. Afterwards, the snow no longer flows even when the slope is increased up to 45° (maximum value allowed by the set-up), which means that just after the liquid-solid transition the material viscosity rapidly increases. Conversely, for slopes larger than approximately 41°, the flow depth typically decreases downstream by roughly 1 cm/m. This corresponds to an *accelerated flow regime*. In that case we observe

that the snow grains start to be fluidized at the end of the flume and the flow appears to be made of a dense layer covered by a cloud of snow grains, a flow type reminiscent of powder snow avalanches.

For intermediate slopes, the flow depth is constant along the channel, so that U is constant also, which corresponds to a *steady and uniform regime*. Various tests resulting in such steady, uniform flows were carried out at different slopes in the range 33-41°, and various depths in the range 4-12 cm (Figure 3 b). Figure 5 shows the typical time evolution of the flow parameters of such a steady, uniform flow, either directly measured or deduced from measurements. After a short transient phase, the three depths along the channel reach similar steady values with some fluctuations around their mean value. This steady state typically lasts for 10 s. The standard deviation of the time fluctuation is around 0.7 cm for each sensor. The normal stress P exerted by the flowing layer on the bottom also reaches a steady value with fluctuations on the order of 10%. The average density of the flow over a cross-section of the channel can be deduced from the simultaneous measurements of P(t) and H(t) using the momentum equation in the direction z: $\bar{\rho} = P/(gH\cos\theta)$. It also reaches a steady-state value with some fluctuations of the order of 10%. Under these conditions the shear stress at the bottom that results from gravity can be deduced from $\bar{\rho}(t)$ and $H(t)$: $\tau_g = \bar{\rho}gH\sin\theta$. We find that the shear stress τ measured at the bottom balances the gravity stress ($|\tau| = \tau_g$), which, using the momentum equation, confirms that the flow is neither accelerated ($|\tau| < \tau_g$) nor decelerated ($|\tau| > \tau_g$), i.e., we have a steady-state, uniform flow.

Figure 4: *Scheme of the three flow regimes as a function of the slope θ, deduced from flow depth measurements along the flume.*

3.2   Variation of the mean velocity U

Figure 6 plots the mean velocity U of several steady and uniform snow flows either as a function of the slope for similar depths, or as a function of depth for the same slopes. The data are remarkably consistent. This is surprising since, due to their continuous metamorphism, the physical characteristics of snow grains differed for all the runs performed at different periods over the 3 years. This suggests that generic, macroscopic properties of dense snow flow can be identified. More precisely, a rheological analysis of these data in terms of flow characteristics only, i.e., without taking into account the different snows, is relevant and should provide some general properties of the snow.

For any simple viscous fluid (Newtonian or power-law), we expect a continuous increase in the average velocity U from zero as the slope angle of the flume is increased. For a yield stress fluid, U is null as long as the slope is lower than a critical value, then continuously increases from zero when the slope is further increased. In contrast, with snow, $U(\theta)$ varies in an unusual way (Figure 6 a): it is equal to zero up to 33°, then abruptly takes a slowly increasing value between $3\,\mathrm{m\,s^{-1}}$ and $4\,\mathrm{m\,s^{-1}}$, for a slope angle between 34° and 41°. This discontinuity in velocity vs slope is reminiscent of the flow instability observed

Figure 5: Time evolution of various quantities during a typical steady uniform flow with $\theta = 37°$: (a) flow depths measurements (averaged values: $H_1(t) \approx 9.8$ cm, $H_2(t) \approx 9.5$ cm, $H_3(t) \approx 9.6$ cm), (b) basal normal stress (P = 170 Pa ± 10%) and deduced average density ($\bar{\rho} = 180\,\mathrm{kg\,m^{-3}} \pm 10\%$), (c) measured basal tangential stress τ (—) and tangential stress deduced from the momentum balance τ_g (gray).

for colloidal suspensions, which somewhat restructure at rest and start to flow abruptly when the slope reaches a critical value (Coussot *et al.*, 2002). It is also reminiscent of the behavior of granular flows over an inclined plane, for which it was shown that no uniform flow could take place at an average velocity smaller than a critical value depending on H (Pouliquen, 1999).

Let us now consider the material behavior in the liquid regime (33-41°) for which steady, uniform flows are observed. Steady and uniform flows down inclined slopes are useful for a rheological investigation, since the variation of the mean velocity U vs flow depth H, i.e., the discharge equation, can be related to the constitutive equation of the flowing material, i.e., the relationship between the shear rate $\dot{\gamma}$ and the shear stress (Astarita *et al.*, 1964; Coussot, 2005). In Appendix A, we recall the demonstration that provides :

$$\dot{\gamma}(\tau_g) = \frac{1}{H} \left.\frac{\partial(UH)}{\partial H}\right|_{\theta}. \tag{1}$$

This result holds for any constitutive law, under the assumptions that the flow is steady and uniform and that the sliding velocity is null.

With snow, the mean velocity slightly increases with the flow depth (Figure 6 b). This evolution is linear in the range 4 cm < H < 12 cm. According to (1), this predicts a decrease in the shear rate $\dot{\gamma}(\tau_g)$ with the inverse of the flow depth H, which corresponds to a material with a resistance to flow decreasing when the shear stress increases. A linear stability analysis, summarized in Appendix B, then strictly demonstrates that no stable flow of a homogeneous material following a behavior in which τ decreases when $\dot{\gamma}$ increases is possible (Coussot, 2005). Hence, the discharge equation does not hold to measure the snow constitutive law. This may be due to a non-null sliding velocity between the snow flow and the bottom. But this may also be the consequence of a heterogeneous flow made of at least two layers with different constitutive equations.

Figure 6: Macroscopic behavior of dense snow flows: mean velocity U for steady and uniform flows performed over 3 years (a) $U(\theta)$ with $H \approx 10$ cm and (b) discharge equation $U(H)$ with $\theta = 35.5°$ and its linear fit: $U = 2 + 15H$ (—).

4 Local behavior

The analysis of the discharge equation is found to be insufficient to determine the constitutive law of the flowing snow, but reveals a possible sliding velocity and/or some structuration of snow flow into layers of different constitutive laws. The measurement of the velocity profile inside the flowing layer enables us to refine this conclusion.

4.1 Velocity profiles

4.1.1 Typical velocity profile and lateral wall effect

For the steady, uniform flows, apart from small fluctuations, the velocity is constant in time. Figure 7 shows a typical velocity profile $V_x(z)$ as measured during our tests. This velocity profile is qualitatively consistent with the limited previous measurements in a small channel (Nishimura & Maeno, 1989; Bouchet *et al.*, 2004), in a large chute (Tiefenbacher & Kern, 2004; Kern *et al.*, 2004) or within real avalanches (Gubler, 1987; Dent *et al.*, 1998).

Note that our measurements correspond to the velocity along a lateral wall (y=10 cm), which might not accurately represents the flow behavior elsewhere if there is a significant shear in the transversal direction. The videos of the free surface allow to track some snow aggregates from five successive pictures corresponding to a displacement of approximately 1 m along the flow direction. Their orthogonal velocity is negligible: $V_y^a \approx 0$. The inset of figure 7 shows the velocity of aggregates V_x^a as a function of their transversal position y. The velocity negligibly varies with y, which demonstrates that transversal shear is negligible. Furthermore, figure 7 shows the good agreement between the velocity measured by optical sensors at y=10 cm and the velocity at the flow surface obtained from height correlation at y=0 cm (see sensor location on Fig.1 b). These results support the assumption that the velocity profile $V_x(z)$ measured

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at the lateral wall represents the behavior well inside the flow.

Figure 7: Velocity profile of a typical snow flow ($\theta = 37°$): each point ($\blacklozenge$) corresponds to the time-averaged measurements from one pair of sensors. The horizontal error bars correspond to the associated standard deviation. The vertical error bars represent the width of a sensor. The highest point ($\lozenge$) was obtained from the correlation between the flow depth measurements taken along the channel. It represents the time-average flow velocity at the free surface, and its vertical error bar corresponds to the standard deviation of the flow thickness over time. The inset shows the velocity V_x^a of aggregates at the free surface as a function of their lateral position y.

4.1.2   Variation of the velocity profiles

Given the possible material transformation from one run to another, the reproducibility of the experiments is a critical point. We therefore repeated some runs keeping constant both the slope and the flow depth. Figure 8 (a) shows that for runs performed during the same night, the resulting velocity profiles are similar, whereas some variation of roughly $\pm15\%$ may be observed for runs performed during nights separated by several weeks (Figure 8 b). This indicates that for a given snow (the same night, i.e., the same series of runs), our initial preparation of the snow is reproducible, and that the grain transformations within the snow cover during a series of runs do not affect the rheological properties of snow, but they may slightly affect the results from one series to another.

Figure 8 (c) shows the velocity profile for runs done during the same night with similar depths but various slopes. Conversely, figure 8 (d) shows the velocity profile for runs performed during the same night with the same slope but various depths. For any slope and any flow depth, the velocity profiles exhibit a common shape: a slightly sheared upper part and a highly sheared lower part, and although there is no velocity measurement at z=0, an extrapolation of the data in the bottom flow region seems to indicate a non-null sliding velocity, approximately 1 ms^{-1}. We observe that the thickness of the lower part remains almost constant, while the thickness of the upper part increases with the flow depth. The critical point is that although the shear stress is assumed to decrease linearly from the flume bottom, the local shear rate (i.e., the slope of the velocity profile) exhibits an abrupt variation at the interface between the two regions (see the inset in figure 8 d). This implies that the flow is heterogeneous: the constitutive law that describes the upper part of the flow cannot describe its lower part. We conclude that

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both a non-null sliding velocity and a heterogeneous behavior explain the inconsistency of the analysis attempting to determine a single constitutive law of snow based on the flow discharge equation (1).

Figure 8: *Variation of the velocity profiles. Reproducibility of the experiment: flows of similar thicknesses (H ≈ 9 cm) and the same slope (37°) (a) performed over the same night, (b) performed over different series a few weeks apart. (c) Effect of the slope: six flows of similar thickness (H ≈ 10.5 cm) performed over one night. (d) Effect of the thickness: three flows at a constant slope (θ = 35.5°) performed over one night; the inset shows the corresponding shear rate at different depths: $\dot{\gamma} = dV_x/dz$.*

4.2 Behavior of each layer

Since the snow flows comprise two layers with different behaviors, we now analyze their rheological properties separately. To this end, we represent the generic form of the velocity profile, a thin, highly sheared basal layer and a much less sheared upper layer, by a simple bilinear function that involves the character-

istic mean shear rates of the top layer ($\dot{\gamma}_u$), the bottom layer ($\dot{\gamma}_b$), and the thickness of the bottom layer (z_b) (Figure 9):

$$V_x(z) = \begin{cases} \dot{\gamma}_b z & \text{if } 0 < z < z_b, \\ \dot{\gamma}_u z + (\dot{\gamma}_b - \dot{\gamma}_u)z_b & \text{if } z_b < z < H. \end{cases} \tag{2}$$

Figure 9: *Typical snow velocity profile with $\theta = 37°$ (●) fitted by the bilinear function of Eqn. 2 (—).*

This simple form obviously does not perfectly describe the velocity profiles. First, it leads to a discontinuity of the shear rate at the interface, whereas the data suggest a very rapid but continuous variation. Secondly, while a linear fit is convincing in the upper part, this is not the same in the lower part: the scarce data can be fitted by a curved profile and there might be some non-zero (sliding) velocity along the bottom. However, in the lower part, data are not enough and not sufficiently precise to satisfactory determine a sliding velocity or a power law for the velocity profile. We therefore settle on describing the behavior of the basal layer using the simple approximation of a linear velocity profile without sliding velocity, and $\dot{\gamma}_b$ represents the averaged shear rate in the bottom layer, including a possible wall effect such as sliding.

Nevertheless, the bilinear function (2) represents the main features of the velocity profiles as a first approximation and thereby makes it possible to further analyze the behavior of each layer. Figure 10 shows the variation of the averaged shear rates in both layers, the thickness of the basal layer as a function of the angle with similar flow depth (left column), or as a function of the flow depth with constant slope (right column). For a given flow thickness, both of the shear rates $\dot{\gamma}_b$ and $\dot{\gamma}_u$ increase when the slope θ is increased. However, they differ by one order of magnitude, even taking into account a sliding velocity of roughly 1 m s^{-1}. Both shear rates slightly decrease when the slope is decreased from 41° to 34°, then abruptly drop to zero for 33°. As a consequence, the shear rate drops from a critical value ($\dot{\gamma}_c$) to zero at a critical slope θ_c, which can be described in a first approximation as: $\theta - \theta_c \propto \dot{\gamma} - \dot{\gamma}_c$.

Moreover, we observe that the thickness z_b of the basal layer slightly decreases when the shear rate $\dot{\gamma}_b$ is increased. For a given slope, the shear rate in the bottom layer ($\dot{\gamma}_b$) increases with the flow depth, as is expected for any homogeneous flowing material. In contrast, the shear rate in the upper layer $\dot{\gamma}_u$ decreases when H increases. Since the average stress in the layer increases with H, this means that the shear rate decreases when the shear stress is increased. According to the stability analysis mentioned in section 3.2 (Appendix B), this implies that the constitutive law of the material in the upper layer evolves when the flow depth is varied.

Figure 10: *Rheological properties of the two layers: (a) $\dot{\gamma}_u(\theta)$, (b) $\dot{\gamma}_u(H)$, (c) $\dot{\gamma}_b(\theta)$, (d) $\dot{\gamma}_b(H)$, (e) $z_b(\theta)$, (f) $z_b(H)$. The data are presented as a function of the series of experiments performed over the same night, i.e., with constant snow grain properties. Flows of similar thickness (left column): $H \approx 8.5$ cm (▲), $H \approx$ 10 cm (●,▼), $H \approx 11.5$ cm (■,♦). Flows of same slope (right column): $\theta = 37°$ (○,△), $\theta = 35.5°$ (□,▽), $\theta = 38°$ (◇). The uncertainty of the velocity measurements is typically $\Delta v \approx 0.5$ ms^{-1}. The uncertainty of the shear rate in the upper layer (thickness ~ 0.1 m) can be estimated by $\Delta \dot{\gamma}_u \sim \Delta v/(0.1m) \sim 5$ s^{-1} and the uncertainty of the shear rate in the basal layer (much thinner : ~ 7 mm) by : $\Delta \dot{\gamma}_b \sim \Delta v/(7mm) \sim 70$ s^{-1}. The estimation of the uncertainty of the interfacial velocity $V_i = v(z_b) = \dot{\gamma}_b z_b$ by Δv makes it possible to deduce the uncertainty of the basal thickness : $\Delta z_b \sim \frac{\Delta V_i}{\dot{\gamma}_b} + \frac{V_i \Delta \dot{\gamma}_b}{\dot{\gamma}_b^2} \sim 2$ mm.*

5 Interpretation in terms of rheological model

Snow flows exhibit unusual characteristics that cannot be described by simple viscoplastic models. Some of these characteristics are reminiscent of those of granular flows, and this is not surprising since snow is made of grains. However, data indicate that snow flows are made of two layers which cannot be described by a single constitutive law. We now suggest an interpretation of such a behavior, based on a shear induced evolution of the snow microstructure. Then we check its plausibility using recent advances in the rheology of granular materials.

5.1 Shear-induced evolution of the snow microstructure

The difference in behavior between the bottom and upper layers may be due to the proximity of the rough bottom which would affect the rheological properties in the basal layer. But it may also be due to a difference in snow microstructure in the two layers. Two visual observations seem to confirm this last interpretation. First, we observe at the end of the flows that several layers of isolated snow grains caught in the asperities of the bed. Secondly, the videos of the free surface reveal the existence of large snow aggregates and so their emerged size can be measured. Figure 11 shows the number N of aggregates of size D throughout a flow.

At the hopper exit, the snow is made of a well-mixed blend of single grains and aggregates of various sizes. Then the two different regions develop as the material moves down the flume. Segregation may lead to the formation of these two layers with different sizes of grains, as was pointed out with cohesionless granular materials (Savage & Lun, 1988; Khosropour *et al.*, 1997; Rognon *et al.*, 2007*a*). However, we observed that the snow aggregates initially at the free surface remain there the entire length of the flume. According to the low shear rate in the upper part of the flow, this suggests a low agitation state that would not favor segregation. But another process can give rise to the formation of these layers, as we observed through simulation of cohesive granular flow (Rognon *et al.*, 2007*b*): the lower regions submitted to the largest stresses tend to liquefy, while the upper regions submitted to the smallest stresses tend to gelify. This mechanism corresponds to a kind of viscosity bifurcation, with some similarity with that observed in soft-jammed systems (Coussot *et al.*, 2002). With dry snow, we may expect that the ice bridges between grains cannot resist the large stresses combined with the grinding process of the bottom roughness, while they are not broken in the upper layer (Fillot *et al.*, 2004). A basal layer made of single grains therefore develops whereas the upper part is made of large aggregates.

5.2 Insights from rheology of granular materials

The composition of snow flow, a mixture of single grains and large aggregates, is reminiscent of a granular material with various grain sizes. It is therefore tempting to compare the snow behavior with that of granular material. Although granular rheology is still a matter of debate, the two last decades of active research brought out many interesting properties. One crucial issue is that granular behavior strongly depends on the solid fraction, and different constitutive laws were proposed to describe either dilute or dense flow. In our situation, the solid fraction within the flow is not measured, but can be roughly estimated as the ratio between the density of the flowing snow (approximately 200 kg/m^3) and the density of aggregates, on the order of the density of the snow cover, about 350 kg/m^3. According to this estimation, the solid fraction of the flowing snow is approximately 0.6, which corresponds to a dense configuration where grains interact through multiple maintained contacts, rather than to a dilute configuration with binary collisions.

Dense flows of grains with no cohesion and with no interstitial fluid have been studied in depth over the last two decades and general knowledge of their rheological properties under various flow conditions has begun to emerge (see for example the review of GdR MiDi, 2004). Although there remains some

Figure 11: *Size distribution of snow aggregates during steady and uniform flows: measurements from video of the free surface of one flow (bars) and power law $\frac{dN}{dD} \propto D^{-2}$ (—) obtained by the correlation of the velocities between two neighboring sensors inside several flows, a method developed by (Bouchet, 2003).*

debate, da Cruz *et al.* (2005) showed that the behavior of monodisperse granular materials in simple shear at moderate velocities is described well using a frictional law, i.e. $\tau = \mu^* P$, in which

$$\mu^* \approx \mu_s^* + b\dot{\gamma}d\sqrt{\rho_p/P}, \tag{3}$$

d is the diameter of grains, ρ_p their density, and μ_s^* and b are two parameters that depend on the grain properties. This constitutive law corresponds to a yield stress fluid whose apparent viscosity is proportional to $d\sqrt{P}$. This was generalized to three-dimensional flows in Jop *et al.* (2006).

It was shown that, when flowing over a rough inclined plane, granular flows exhibit the same flow regimes as snow: they are steady and uniform in a wide range of slopes, above which they accelerate and below which they decelerate and stop. For steady uniform flows, the stress distribution is of the hydrostatic type as long as the solid fraction is constant. As a consequence, the effective friction coefficient μ^* is simply related to the slope: $\mu^*(z) = \tan\theta$ and is constant. Integrating equation (3) we find the following shear rate distribution, equivalent to the one obtained with the Norem-Irgens-Schieldrop rheology (Norem *et al.*, 1987):

$$\dot{\gamma}(z) \propto \frac{\tan\theta - \mu_s^*}{d}\sqrt{(H - z)\cos\theta}. \tag{4}$$

This behavior is sufficient to qualitatively predict the behavior of each layer when considering that they are made of different grain sizes. Let us consider that the bottom layer of the snow flow is made of single snow grains (d≈0.4 mm). Then the granular behavior (4) predicts the behavior of this layer well, i.e., the increase in the shear rate $\dot{\gamma}_b$ with both the slope θ and the flow depth H. Let us now consider that the size of the grains in the upper layer is given by the size of the aggregates (d≈ 1 cm). Since the granular behavior (4) predicts that the shear rate scales with the inverse of the grain size: $\dot{\gamma} \propto 1/d$ (Rognon *et al.*, 2007*a*), it predicts the drop of one order of magnitude of the shear rate between the two layers. Last, we expect that the flow thickness limits the maximal size of the aggregates in the upper layer. We therefore can consider that the typical size of the grains in this layer increases when H is increased. Then equation (4) predicts the surprising decrease in the shear rate in this layer ($\dot{\gamma}_u$) for an increase in the flow depth H.

Despite this qualitative agreement, fitting the snow velocity profile by (4) requires two additional difficulties to be overcome. First, it requires to know the evolution of the typical diameter of grains along the flow depth, which certainly evolves through the upper layer, as suggested in (Rognon *et al.*, 2007*a*). Then, the effect of the rough bottom on the behavior of snow in the bottom layer must be understood. It was shown that the constitutive frictional law (3) does not describe granular flow near the wall. The perturbation due to the basal roughness is still not completely understood, but da Cruz *et al.* (2005); Rognon *et al.* (2007*a*) pointed out an increase in the shear rate in a region approximately five grains thick. In our snow flow, this would correspond to a basal layer approximately 1 mm thick, where the shear rate would be strongly increased.

6   Conclusion

In this paper we investigated the rheological behavior of a dense flow of dry snow through in-situ chute flow experiments. Despite substantial experimental difficulties, an extensive series of steady flows were performed to explore wide ranges of slope and flow discharge. This unique data set enabled us to contribute new insights into the rheology of dry snow.

The most surprising observation is that the characteristics of flows performed at different periods over 3 years are remarkably consistent. This means that certain generic rheological properties of dry snow could be identified without taking into account the variation of snow grain properties due to the continuous snow metamorphism. We first identified three flow regimes as a function of the slope: flows are decelerated below approximately 33°, accelerated above approximately 41°, and are uniform between these two limits. The mean velocity of the flow is mostly constant around 4 ms^{-1} for any uniform flow, but drops abruptly to zero when approaching the critical slope 33°. This evolution differs from that of simple yield stress fluids, but is reminiscent of the behavior of granular materials. We then analyzed the velocity profiles within uniform snow flows, which were found to exhibit a typical shape: highly sheared in a thin basal layer and much less sheared in the thick top layer. The drop in the shear rate at the interface where stresses vary slightly indicates that the constitutive law of snow must be different in each layer. We interpret this heterogeneity to be a consequence of a shear-induced evolution of the snow's microstructure, which gives rise to different materials within the lower and the upper parts of the flow. Our interpretation is that the snow flow is initially made of a well-mixed blend of single grains and aggregates, but solid bridges between grains cannot resist in the lower region submitted to larger stresses combined with the grinding process of the basal roughness. Consequently, a basal layer of single grains develops whereas the aggregates are not broken in the upper parts. Moreover, sintering can occur at a low shear rate, which would enhance the heterogeneity of the materials between layers: new bridges can form in the slowly sheared upper part, but cannot form in the rapidly sheared lower part. This structuration - a low viscous basal layer ensuring fast motion of the solid-like (or highly viscous) upper region representing most of the material volume - is reminiscent of a caterpillar and should contribute to the potential of snow avalanches to destroy obstacles such as dams or breaking mounds.

Recent advances in the formulation of constitutive laws describing the dense flow of cohesionless grains seems to confirm this interpretation of a two-layered flow. Although granular rheology is still a matter of debate, it was shown that the apparent viscosity of the material is proportional to the size of the grains. This constitutive law predicts a substantial difference in shear rate between the basal layer of single grains and the upper layer made of aggregates, and it also qualitatively predicts the behavior that we measured in each layer. However, a quantitative description of the snow velocity profiles using granular behavior requires further studies to understand how the aggregates are distributed along the flow depth, and how the flow is affected near the basal roughness. Nevertheless, these results support the approaches consisting in modeling snow behavior using experimental granular flows. They also point out the crucial role of intergranular cohesive forces in snow behavior through the formation of aggregates within the flow, and therefore suggest that experiments with cohesive grains such as wet glass beads or powders, or with

polydispersed grains, would be more appropriate to model dense snow flow.

This study focused on dense flow of dry snow, but few experiments were performed using wet or fresh snow. In both cases, flows stop in the channel even when the slope is increased up to 45° (maximum value allowed by the set-up). This difference requires further studies of the behavior of wet and fresh snow, which are often involved in real avalanches. Another process to investigate is the fluidization that is observed at the surface of accelerated flow. It seems to be the first stage of the development of powder avalanches, whose behavior is also a promising field of research.

Acknowledgments

A   Discharge equation and constitutive law

In this appendix, we recall the demonstration that allows to deduce the constitutive law of a material from flows down a slope. It was previously introduced in Astarita *et al.* (1964); Coleman *et al.* (1996). Let us consider a material described by its constitutive law which relates the shear rate to the shear stress, $\dot\gamma(\tau)$. When a layer of thickness H of this material is flowing down a slope, the velocity profile is given by $V_x(z) = V_x(0) + \int_0^z \dot\gamma(\tau(\xi))d\xi$. Under the assumption that the sliding velocity $V_x(0)$ is null, the flow rate per unit of width reads $Q = UH = \int_0^H \left[\int_0^z \dot\gamma(\tau(\xi))d\xi\right] dz$, and, using an integration by part, $Q = \int_0^H \dot\gamma(\tau(z))(H - z)dz$. Under the assumption that the flow is steady and uniform, the shear stress profile is known, $\tau(z) = \tau_g(1 - \frac{z}{H})$ where $\tau_g = \bar\rho gH \sin\theta$ is the basal shear stress. Applying a change of variable on the last integral, $z \to H(1 - \tau/\tau_g)$, it becomes $Q = -\frac{H^2}{\tau_g^2} \int_{\tau_g}^0 \dot\gamma(\tau)\tau d\tau$. We now take the partial derivative of Q with respect to H, $\frac{\partial Q}{\partial H} = \frac{\partial Q}{\partial \tau}\cdot\frac{\partial \tau}{\partial H} = \frac{\partial \tau}{\partial H}\frac{H^2}{\tau_g^2} \int_0^{\tau_g} \left[\frac{\partial\dot\gamma(\tau)}{\partial\tau}\tau + \dot\gamma(\tau)\right] d\tau$. With $\frac{\partial \tau}{\partial H} = \frac{\tau_g}{H}$ and using an integration by part for the first term of the integral, we get $\frac{\partial Q}{\partial H} = \frac{H}{\tau_g} \left[\dot\gamma(\tau)\tau\right]_{\tau=0}^{\tau_g} - \int_0^{\tau_g} \dot\gamma(\tau)d\tau + \int_0^{\tau_g} \dot\gamma(\tau)d\tau$. Consequently, we can use the channel as a rheometer since the constitutive law of the flowing material, $\dot\gamma(\tau)$, is given by the variation of the flow rate with respect to the flow thickness, at a constant slope:

$$\dot\gamma(\tau_g) = \frac{1}{H} \left.\frac{\partial(UH)}{\partial H}\right|_\theta.$$

Note that this result holds under the assumption that the flows are steady and uniform and that there is no sliding velocity.

B   Linear stability

Let us assume for the sake of simplicity that a constant (apparent) shear rate $\dot\gamma$ is imposed on the material. Thus the velocity under stable conditions is $v_0(z) = \dot\gamma z$. We now assume that the velocity field is slightly perturbed so that its expression becomes $v(z) = v_0(z) + v_1 \exp(ikz + \omega t)$, in which k and ω are two

(real) parameters and v_1 is small compared to v_0. Under these conditions, assuming negligible gravity and normal stress effects, the momentum equation along the flow direction reduces to $\rho \frac{\partial v(z)}{\partial t} = \frac{\partial \tau}{\partial z}$. Since we can write $\frac{\partial \tau}{\partial z} = \frac{\partial \dot{\gamma}}{\partial z} \frac{\partial \tau}{\partial \dot{\gamma}} = \frac{\partial^2 v}{\partial z^2} \frac{\partial \tau}{\partial \dot{\gamma}}$, the momentum equation becomes: $\rho\omega = -k^2 \frac{\partial \tau}{\partial \dot{\gamma}}$. This implies that if τ decreases when $\dot{\gamma}$ increases, ω is positive, which means that the amplitude of the perturbation, proportional to $\exp(\omega t)$, constantly increases with time. Since a real perturbation (of any form) can be broken down as the sum of an infinity of sinusoidal perturbations of the above form, each of them leading to flow instability, the flow is unstable. When such a constitutive equation (with decreasing stress) is observed in practice it necessarily reflects a localization of shear in a thickness on the order of few diameters of material elements, and the apparent behavior in fact reflects the local behavior of this thin layer for which the validity of the continuum assumption is doubtful.

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