Friction law in dense granular flows ⋆

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Abstract

The understanding of the dense granular flow regime (intermediate between quasi-static and collisional regimes) has recently progressed significantly. Discrete numerical simulations have provided detailed informations on the flow of assemblies of disks in various geometries (homogeneous plane shear, annular shear and inclined plane). In the case of cohesionless quasi mono-dispersed and rigid grains, the analysis of the dependencies of the effective friction coefficient, ratio of shear to normal stress, on the shear state (defined by the shear rate and pressure) and on the mechanical characteristics of the material has made possible the formulation of a local constitutive law. This has shown the crucial role of a dimensionless parameter, called inertial number, which rules the friction law in the dense regime. This law remains true in the case of bi-dispersed flows, as early as the average diameter of the grains in taken into account. It is also true in the case of cohesive grains, once a second dimensionless number, characterizing the intensity of cohesion, is introduced.

Key words: Rheology, Granular Flows, Friction, Cohesion, Discrete numerical simulations, Polydispersity

1. Introduction

The understanding of granular flows is a major challenge both in geophysics and in industry [1,2,3]. A major goal of the rheophysical studies is to evaluate the rheological laws as well as their physical origin at the scale of the grains and of their interactions [4,5]. We restrict our analysis to assemblies of grains interacting through direct contacts [6] (neglecting the influence of the interstitial fluid), and to dense flows, an intermediate regime between quasi-static deformations of soil mechanics [4] and rapid and dilute flows, which can be described by the kinetic theory of dense gases [7]. Such dense flows of dry grains have been actively studied during the last decade [8,9]. We shall insist here on the results obtained through discrete numerical simulations [6], which have been confronted to experimental measurements on model materials. We shall first describe our general approach. Then we shall explain the friction law measured in various geometries for cohesionless quasi mono-dispersed grains. We shall then discuss the influence of polydispersity and cohesion. In conclusion, we shall emphasize the subjects on which the research now focuses.

2. General approach

Discrete numerical simulations (we have used standard molecular dynamics methods) [6,10] provide microscopic informations, at the level of the contact network, difficult to access experimentally. The problem has been studied in two-dimensional geometry, on an assembly of disks of average diameter d and areal mass ρ_g ($m = \rho_g \pi d^2/4$). The contact law between grains is described by a normal elastic stiffness k_n (assuming linear elasticity), a normal viscous damping parameter associated to the normal restitution coefficient e in binary collisions, and a coulombian friction coefficient μ. In a first step, the grains are cohesionless and slightly polydispersed ($\pm 20\%$). Our general approach consists in applying a shear to the material, and to measure locally the velocity u, the shear rate $\dot{\gamma}$, the solid fraction ν and the components of the stress tensor, the shear stress τ and the pressure P (we observe that the two normal stress components are nearly equal). We have studied three geometries. The first one (Fig.1a) is the plane shear in the absence of gravity with or without walls. This is the simplest one since then the state of the material is homogeneous (except near the walls). We control the shear rate and the pressure. We note x the flow direction and y the orthogonal

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direction. Periodic boundary conditions are applied along x (the length of the system is of the order of fifty grains) and along y without walls (the width H of the system is a few tens of grains). The second geometry (Fig.1b) is the annular shear. The material is confined between an inner cylinder of radius R_i which has a rotation velocity Ω and an outer cylinder of radius R_e which applies a radial presssure P. We note r and θ the radial and orthoradial directions. Periodic boundary conditions are applied along θ [11]. The pressure is approximately constant along r, while the shear stress decreases as $1/r^2$. The third geometry (Fig.1c) is the inclined plane. A layer of height H flows down a slope θ under the influence of gravity g. We note x the flow direction, along which periodic boundary conditions are applied, and y the orthogonal direction ($y = 0$ corresponds to the bottom). In those three geometries, the walls must be sufficiently rough to exclude sliding velocity. This roughness is made of contiguous grains sharing the same geometrical and mechanical characteristics than the flowing grains. We shall not describe in this paper the particular behavior of the material in the vicinity of the walls (structuration in layers), nor the influence of the roughness scale which rules the friction at the wall and the possible sliding [11]. We only consider steady (independent of time) and uniform (independent of x or θ) flows, so that space and time averages are performed. In the first geometry, an average along y is also performed (taking apart the first layers near the walls). In the two other geometries, we measure the dependencies along r or y of the various quantities.

3. Friction law and inertial number

In the limit of rigid contacts, there remain two characteristic times in the problem : the inertial one $\tau_i = d\sqrt{\rho_g/P}$ and the shear one $\tau_c = 1/\dot{\gamma}$, from which it is natural to define a dimensionless parameter, called inertial number I:

$$I = \frac{\tau_i}{\tau_c} = \dot{\gamma} d \sqrt{\frac{\rho_g}{P}}. \tag{1}$$

We expect that the flow properties essentially depend on I, except for possible other dependencies associated to the mechanical and geometrical characteristics of the material. The value of I allows to classify the flow regimes. As I increases, the average contact time decreases and the material dilates. For $I \leq 10^{-3}$, the material tends to the quasi-static regime, in which it is decribed as an elasto-plastic solid (the so-called critical state of soil mechanics) [4]. For $I \geq 10^{-1}$, it approaches the collisionnal regime, where it may be described through extension of the kinetic theory of dense gazes to dissipative particles [7]. The intermediate regime ($10^{-3} \leq I \leq 10^{-1}$) corresponds to dense flows, where the material is above its flow threshold, in a liquid rather than gazeous state. The grain motions are strongly correlated and the assumption of uncorrelated binary collisions used for dilute gazes is clearly inappropriate. There exists a percolating contact network, strongly fluctuating both in space and time. In the following, we shall be interested in the effective friction coefficient of the material, a local quantity defined by $\mu^* = \tau/P$. In the plane shear geometry [10], we could evidence a robust *friction law*, with the approximate following expression (Fig.2) :

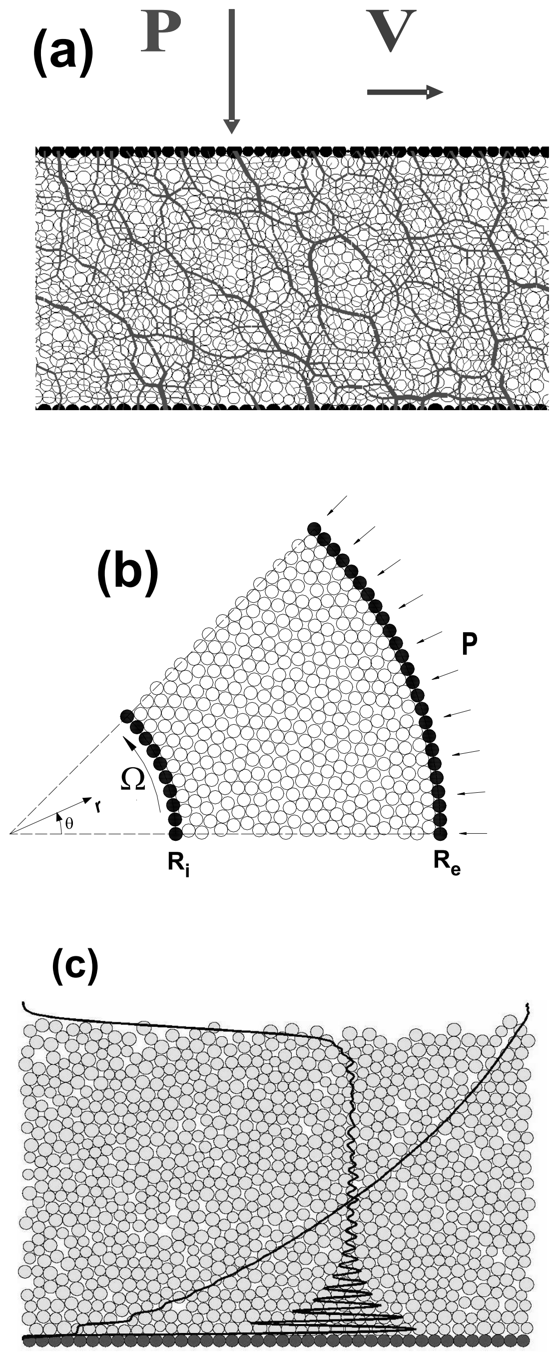


Figure 1. Flow geometries : (a) Plane shear between two rough walls (with the network of normal forces), (b) Annular shear, (c) Inclined plane (with the velocity and solid fraction profiles).

$$\mu^*(I) \simeq \mu^*{}_{min} + bI. \tag{2}$$

This means that the friction coefficient has a minimum value $\mu^*{}_{min}$ in the quasi-static regime (which can be identified to the internal friction in the critical state), increases approximately linearly in the dense regime, and seems to saturate in the collisional one. The shear stress is then the sum of a contribution associated to the Coulomb law in the plastic state, and of a viscous contribution associated to collisions, with a viscosity unusually proportional to the square root of the pressure : $\tau \simeq {\mu^*}_{min} P + b\sqrt{\rho_g P}\dot{\gamma}$. A granular material in the dense flow regime then appears as a visco-plastic fluid. A complementary information is the *dilatancy law*, which describes the variations of the solid fraction as a function of I [10]: when I increases, ν slowly decreases, approximately linearly, from its maximum value. In the rigid contact limit, apart from the extreme cases of e close to 1 and/or μ close to 0, this constitutive law very slightly depends on the mechanical characteristics of the grains.

In the annular shear geometry [11,13], we have shown that the same friction law is observed in the dense regime (Fig.2). This provides a first explanation of the shear localization. But, in this geometry, μ^* decreases as $1/r^2$ from its value at the inner cylinder, and consequently becomes smaller than ${\mu^*}_{min}$ far from it (possibly from $r = R_i$ if $\Omega d\sqrt{\rho_g/P}$ is sufficiently small). The material is then in the quasi-static regime, and the shear localization may be explained by a different friction law, akin to creep flow activated by the fluctuations near the rough wall.

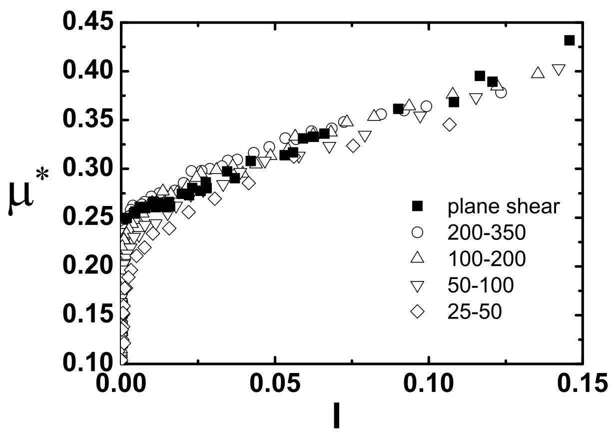


Figure 2. Friction law in plane shear and annular shear geometries for various $(R_i - R_e)$.

The friction law which has been identified in homogeneous shear may be used to predict the shear rate profile within a steady dense flow down an inclined plane, for which $\mu^* = \tan\theta$, which fixes the inertial number I and the mass density $\rho = \rho_g \nu$ inside the layer (Fig.1c). The pressure is thus hydrostatic : $P(y) = \rho_g \cos\theta (H - y)$. Approximating $\tan\theta \simeq \theta$ and $\cos\theta \simeq 1$ and identifying μ^* to $\tan\phi \simeq \phi$, the threshold flowing angle, it comes :

$$\dot{\gamma}(y) \simeq \frac{\sqrt{\nu g}\,(\theta - \phi)}{bd}\sqrt{H - y}. \tag{3}$$

Taking into account the absence of sliding near the rough base, one deduces the velocity profile and eventually the *flow law* relating the average velocity V to the inclination θ and to the height H of the flow :

$$V(\theta, H) \simeq \frac{2}{5}\frac{\sqrt{\nu g}}{bd}\,(\theta - \phi)\, H^{3/2}. \tag{4}$$

Discrete numerical simulations [12,13,14,15] as well as experiments [13,16,17] of flows down rough inclines are in good agreement with those predictions, but is is furthermore observed that the threshold angles (jamming or unjamming) depend on the height [12,13,14,17]. The study of a more complex situation, the spreading of a granular mass down a rough slope [18], has revealed an impressive comparison between the experiment and the numerical calculation through the Saint Venant approach, when taking into account the previous friction law (2). Another significant result is the crucial influence on the flow of the frictional lateral walls [19,20]: when the flow rate in a channel is increased, a solid heap forms on which a superficial layer flows. The slope of this heap and the height of the flowing layer depend on the ratio of the two friction coefficients, internal and of the lateral walls, and on the width of the channel. More detailed experimental studies, varying the friction of the lateral walls, have shown that a three-dimensional tensorial formulation of the friction law (2) is able to predict the full three-dimensional velocity profile inside the channel [20].

4. Influence of polydispersity

In real flows, the granulometry is often large. The consequences on the rheological laws are not yet well understood. We have studied the flow of a bi-disperse mixture down an inclined plane [21], once a steady segregated state is obtained. The material is a bi-disperse assembly of disks of the same mass density : n_1 small disks of average diameter d_1 and n_2 large disks of average diameter d_2. This mixture is characterized by the size ratio $D = d_2/d_1$ and by the areal proportion of large grains $S = (n_2 d_2^2)/(n_2 d_2^2 + n_1 d_1^2)$. The roughness is made of neighboring small grains. Steady flows are observed in a range $\theta - H$ which depends on the mixture. The grains are organized in three layers: small grains near the rough base, large grains near the free surface and a mixture in the middle. We systematically observe a decrease of the shear rate in the upper part of the flow when S and/or D increase. Furthermore, when $S \geq 3/4$ and $D \geq 3$, we observe a strong increase of the shear rate near the base. We relate this shear localization to the segregation of the material. Considering Eq. 3, for given inclination θ and height H, the shear rate must decrease as the inverse of the grain diameter. More precisely, we have measured locally, as a function of y, the effective friction coefficient μ^* and the inertial number $I_{<d>}$ obtained by considering the average local size $< d >$ of the grains. The plot of μ^* as a function of $I_{<d>}$ drawn on Fig.3 shows that the friction law (2) has the same qualitative shape in the case of mixtures (but the parameters ${\mu^*}_{min}$ and b are affected by the composition of the mixture and by its interaction with the roughness [11,22]):

$$\mu^*(I_{<d>}) \simeq {\mu^*}_{min} + bI_{<d>}. \quad (5)$$

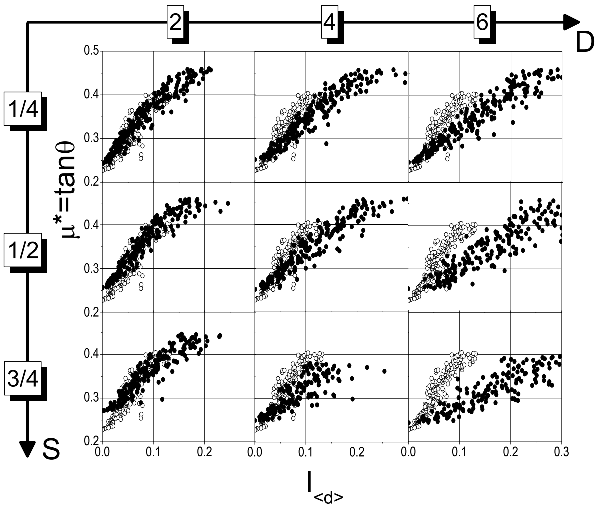


Figure 3. Friction law in a binary mixture as a function of S and D: (∘) monodispersed case (±20%), (black ∘) bidispersed case.

5. Influence of cohesion

In various industrial and geophysical situations, cohesion forces between grains cannot be neglected anymore. They are usually classified in three groups according to their physical origin : capillary forces [23], solid bridges [24], and direct adhesion between the grain surfaces associated to van der Waals forces, like in powders [25]. This cohesion strongly affects the mechanical properties of a granular material [2,3]. The microstructure of a cohesive piling is extremely sensitive to its preparation. The sample is more or less heterogeneous due to the formation of clusters and this loose structure is evidenced in plastic flows [26] or during the compaction of the sample [27]. The cohesion also strongly increases the flow threshold. But the dense flow regime is still not well understood. In the case of very small particles like powders, because of the strong influence of the interstitial fluid, the granular material transits directly from a solid to a suspension of fragile clusters [28]. However this dense regime has been observed experimentally in dense snows made of grains of a few hundred microns [29] or with wet glass beads [30]. We have studied the flow of cohesive grains in the dense regime [31,32], following the work of other research teams [33,34,35,36,37], but with an emphasis on the quantitative determination of the rheological laws.

The cohesion models add to the usual repulsion force an attractive force $N^a(h)$, function of the normal deflection of the contact h, and which shape depend on the physical origin of the cohesion. We note $N(h) = N^e(h) + N^a(h)$ the total static normal force, N^c the maximum attractive force and h^c the deflection at equilibrium (for which $N(h^c) = 0$). The range of the attractive force is assumed null, and we do not take into account any hysteresis. We have chosen a simple cohesion model which takes into account the main characteristic of cohesion models, that is to say the maximum attractive force N^c : $N^a(h) = -\sqrt{4k_n N^c h}$. This leads to introduce a second dimensionless number η which measures the cohesion intensity, ratio of the maximum attractive force on the average normal force due to pressure (in 2D):

$$\eta = \frac{N^c}{Pd}. \quad (6)$$

This parameter is equivalent to other dimensionless numbers proposed in the literature [27,34], like the granular Bond number $Bo_g = N_c/(mg)$ in presence of gravity.

In the homogeneous shear geometry, we have measured the dependencies of μ^* as a function of the two dimensionless numbers I and η (Fig.4a). We have observed that the friction law (2) may be extended to cohesive grains :

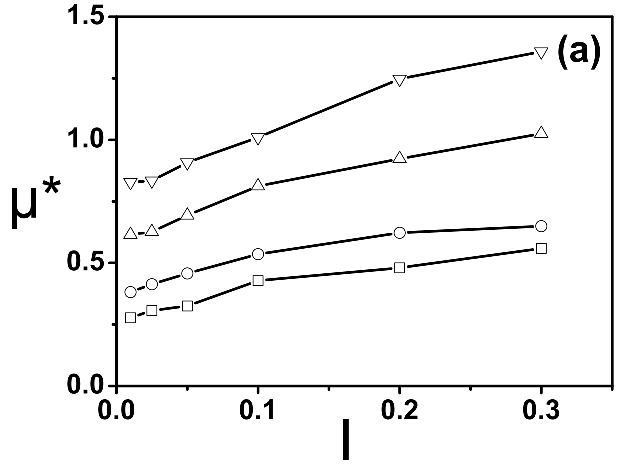
$$\mu^*(I, \eta) \simeq {\mu^*}_{min}(\eta) + b(\eta)I. \quad (7)$$

The two functions ${\mu^*}_{min}(\eta)$ and $b(\eta)$ have the same shape (Fig.4b) : they strongly increase with the cohesion intensity.

For a flow down an inclined plane, the pressure increases with the depth z measured from the free surface and the cohesion intensity strongly increases near the free surface: $\eta(z) \simeq Bo_g/(z/d)$. We thus predict the presence of a superficial plugged layer where the shear rate drops to zero, with a width $\simeq (Bo_g/10)d$, while the deep layer keeps a non cohesive behavior. This effect, significant for $Bo_g \geq 10$, is indeed observed in discrete numerical simulations [38,32] and could be evidenced experimentally using small enough glass beads ($d \leq 1$ mm) and water as the interstitial fluid.

6. Conclusion

In this short paper, we have not described the measurement of other quantities accessible to discrete numerical simulations (organization of the contact network, correlations, fluctuations. . .), the comparison with experimental studies, or other phenomena such as the fluid-solid transition (jamming or unjamming). We conclude on the directions on which research is progressing : the account of three-dimensional geometry, the study of more complex materials (shape of the grains, more realistic cohesion models, larger granulometric distribution, influence of an interstitial fluid. . .), and of more complex flow geometries (interaction with obstacles, mixing. . .).



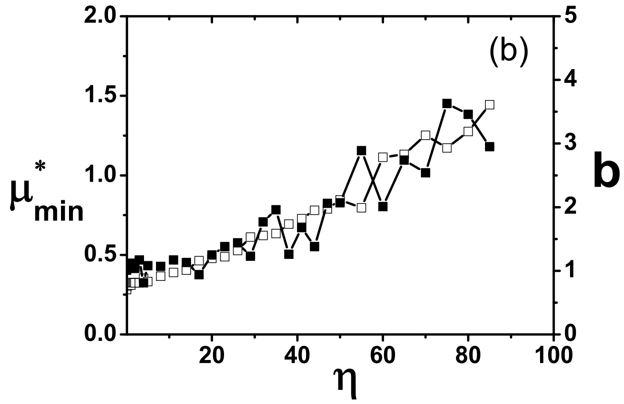


Figure 4. Friction law of cohesive grains : (a) $\mu^*(I)$ for $\eta = 0$ (□), $\eta = 10$ (○), $\eta = 30$ (△) and $\eta = 50$ (▽), (b) ${\mu^*}_{min}(\eta)$ (□) and $b(\eta)$ (■).

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